

# Chapter 10: Key Management

---

- Session and Interchange Keys
- Key Exchange
- Key Generation
- Cryptographic Key Infrastructure
- Storing and Revoking Keys
- Digital Signatures

# Overview

---

- Key exchange
  - Session vs. interchange keys
  - Classical, public key methods
  - Key generation
- Cryptographic key infrastructure
  - Certificates
- Key storage
  - Key escrow
  - Key revocation
- Digital signatures

# Notation

---

- $X \rightarrow Y : \{ Z \parallel W \} k_{X,Y}$ 
  - $X$  sends  $Y$  the message produced by concatenating  $Z$  and  $W$  enciphered by key  $k_{X,Y}$ , which is shared by users  $X$  and  $Y$
- $A \rightarrow T : \{ Z \} k_A \parallel \{ W \} k_{A,T}$ 
  - $A$  sends  $T$  a message consisting of the concatenation of  $Z$  enciphered using  $k_A$ ,  $A$ 's key, and  $W$  enciphered using  $k_{A,T}$ , the key shared by  $A$  and  $T$
- $r_1, r_2$  nonces (nonrepeating random numbers)

# Session, Interchange Keys

---

- Alice wants to send a message  $m$  to Bob
  - Assume public key encryption
  - Alice generates a random cryptographic key  $k_s$  and uses it to encipher  $m$ 
    - To be used for this message *only*
    - Called a *session key*
  - She enciphers  $k_s$  with Bob's public key  $k_B$ 
    - $k_B$  enciphers all session keys Alice uses to communicate with Bob
    - Called an interchange *key*
  - Alice sends  $\{ m \}_{k_s} \{ k_s \}_{k_B}$

# Benefits

---

- Limits amount of traffic enciphered with single key
  - Standard practice, to decrease the amount of traffic an attacker can obtain
- Prevents some attacks
  - Example: Alice will send Bob message that is either “BUY” or “SELL”. Eve computes possible ciphertexts  $\{ \text{“BUY”} \} k_B$  and  $\{ \text{“SELL”} \} k_B$ . Eve intercepts enciphered message, compares, and gets plaintext at once

# Key Exchange Algorithms

---

- Goal: Alice, Bob get shared key
  - Key cannot be sent in clear
    - Attacker can listen in
    - Key can be sent enciphered, or derived from exchanged data plus data not known to an eavesdropper
  - Alice, Bob may trust third party
  - All cryptosystems, protocols publicly known
    - Only secret data is the keys, ancillary information known only to Alice and Bob needed to derive keys
    - Anything transmitted is assumed known to attacker

# Classical Key Exchange

---

- Bootstrap problem: how do Alice, Bob begin?
  - Alice can't send it to Bob in the clear!
- Assume trusted third party, Cathy
  - Alice and Cathy share secret key  $k_A$
  - Bob and Cathy share secret key  $k_B$
- Use this to exchange shared key  $k_s$

# Simple Protocol

---

Alice  $\xrightarrow{\{ \text{request for session key to Bob} \} k_A}$  Cathy

Alice  $\xleftarrow{\{ k_s \} k_A \parallel \{ k_s \} k_B}$  Cathy

Alice  $\xrightarrow{\{ k_s \} k_B}$  Bob



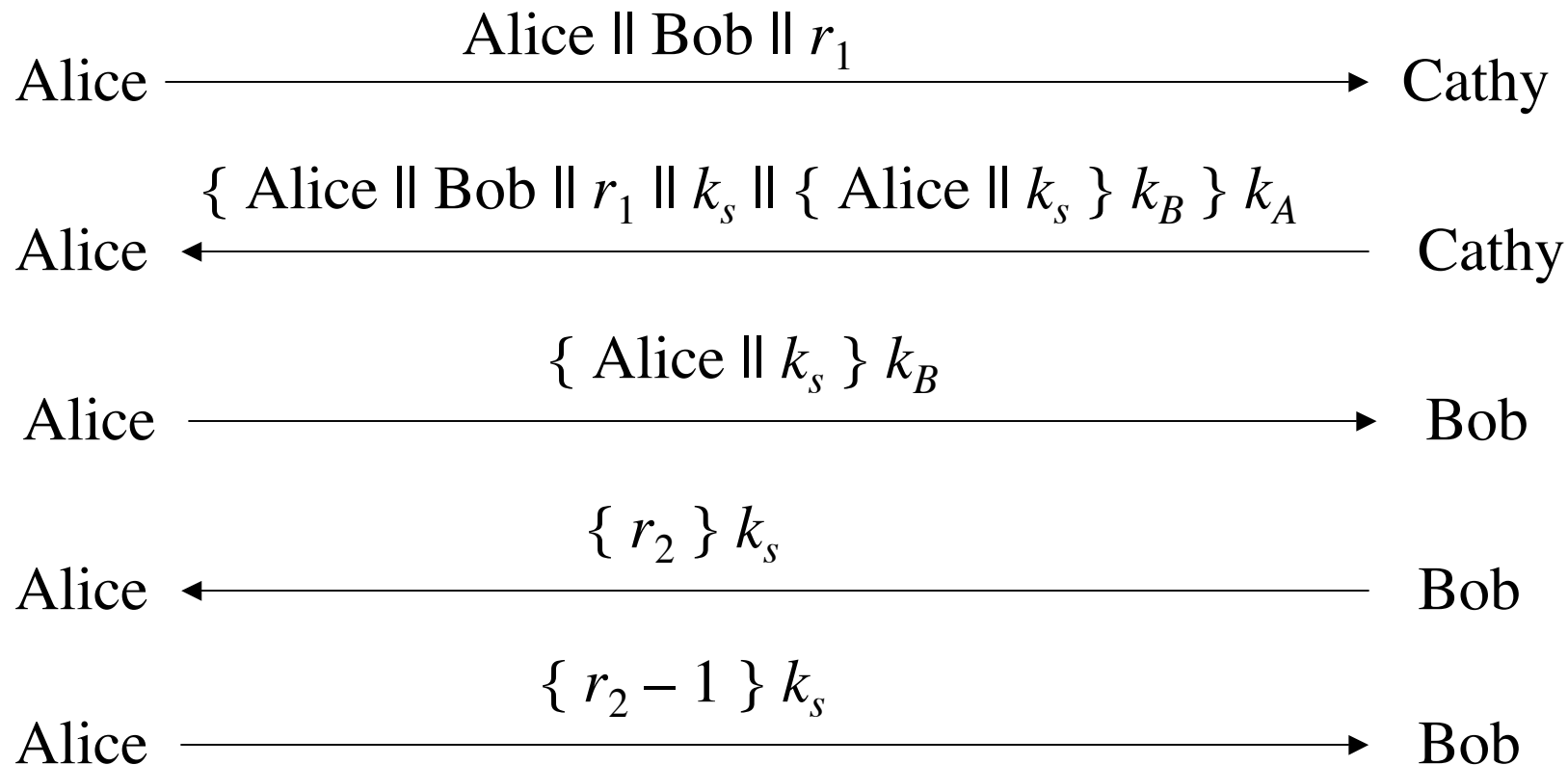
# Problems

---

- How does Bob know he is talking to Alice?
  - Replay attack: Eve records message from Alice to Bob, later replays it; Bob may think he's talking to Alice, but he isn't
  - Session key reuse: Eve replays message from Alice to Bob, so Bob re-uses session key
- Protocols must provide authentication and defense against replay

# Needham-Schroeder

---



# Argument: Alice talking to Bob

---

- Second message
  - Enciphered using key only she, Cathy knows
    - So Cathy enciphered it
  - Response to first message
    - As  $r_1$  in it matches  $r_1$  in first message
- Third message
  - Alice knows only Bob can read it
    - As only Bob can derive session key from message
  - Any messages enciphered with that key are from Bob

# Argument: Bob talking to Alice

---

- Third message
  - Enciphered using key only he, Cathy know
    - So Cathy enciphered it
  - Names Alice, session key
    - Cathy provided session key, says Alice is other party
- Fourth message
  - Uses session key to determine if it is replay from Eve
    - If not, Alice will respond correctly in fifth message
    - If so, Eve can't decipher  $r_2$  and so can't respond, or responds incorrectly

# Denning-Sacco Modification

---

- Assumption: all keys are secret
- Question: suppose Eve can obtain session key.  
How does that affect protocol?
  - In what follows, Eve knows  $k_s$

Eve  $\xrightarrow{\{ \text{Alice} \parallel k_s \} k_B}$  Bob

Eve  $\xleftarrow{\{ r_2 \} k_s}$  Bob

Eve  $\xrightarrow{\{ r_2 - 1 \} k_s}$  Bob

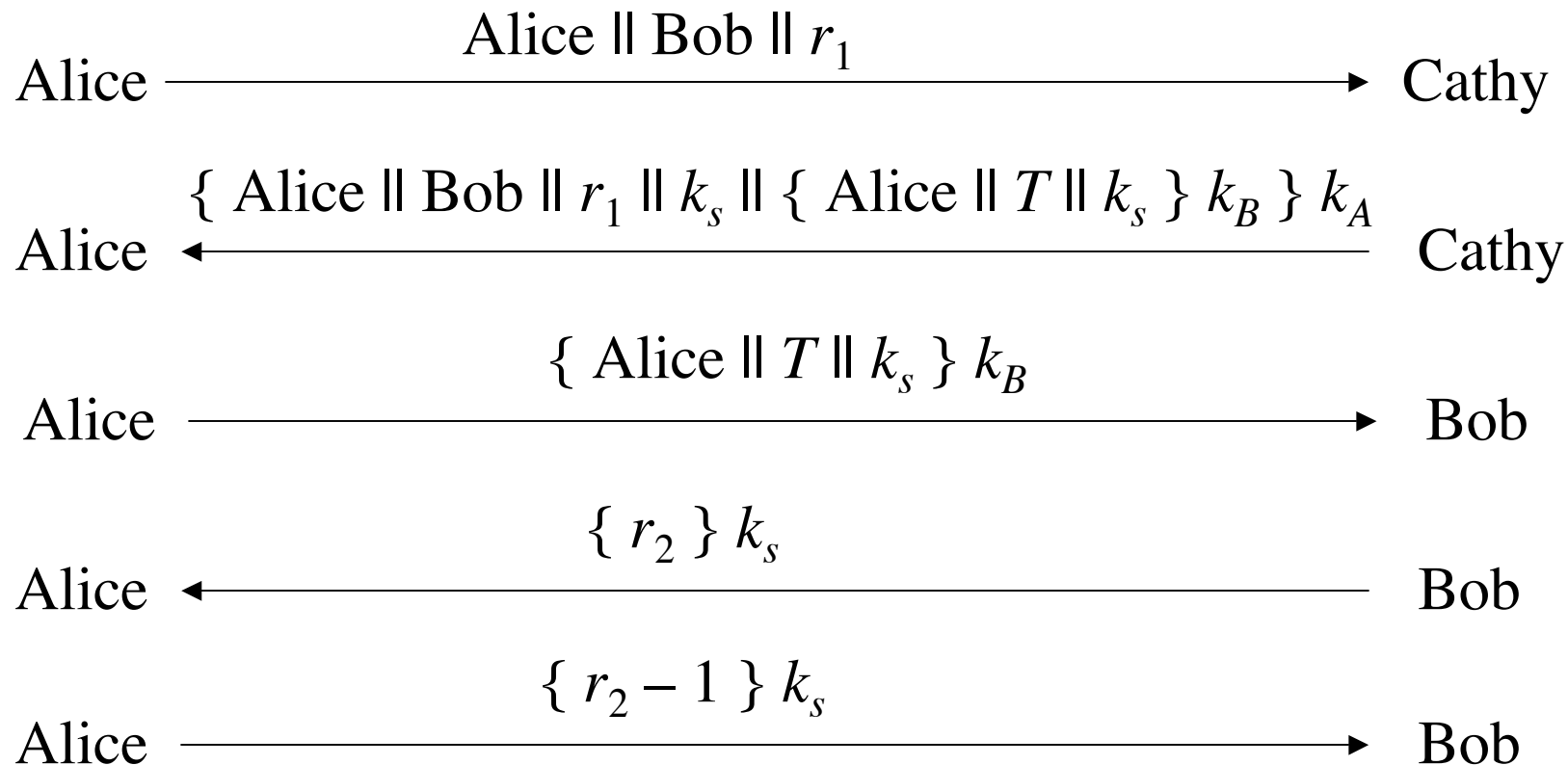
# Solution

---

- In protocol above, Eve impersonates Alice
- Problem: replay in third step
  - First in previous slide
- Solution: use time stamp  $T$  to detect replay
- Weakness: if clocks not synchronized, may either reject valid messages or accept replays
  - Parties with either slow or fast clocks vulnerable to replay
  - Resetting clock does *not* eliminate vulnerability

# Needham-Schroeder with Denning-Sacco Modification

---



# Otway-Rees Protocol

---

- Corrects problem
  - That is, Eve replaying the third message in the protocol
- Does not use timestamps
  - Not vulnerable to the problems that Denning-Sacco modification has
- Uses integer  $n$  to associate all messages with particular exchange



# The Protocol

---

Alice  $\xrightarrow{n \parallel \text{Alice} \parallel \text{Bob} \parallel \{ r_1 \parallel n \parallel \text{Alice} \parallel \text{Bob} \} k_A}$  Bob

Cathy  $\xleftarrow{n \parallel \text{Alice} \parallel \text{Bob} \parallel \{ r_1 \parallel n \parallel \text{Alice} \parallel \text{Bob} \} k_A \parallel \{ r_2 \parallel n \parallel \text{Alice} \parallel \text{Bob} \} k_B}$  Bob

Cathy  $\xrightarrow{n \parallel \{ r_1 \parallel k_s \} k_A \parallel \{ r_2 \parallel k_s \} k_B}$  Bob

Alice  $\xleftarrow{n \parallel \{ r_1 \parallel k_s \} k_A}$  Bob

# Argument: Alice talking to Bob

---

- Fourth message
  - If  $n$  matches first message, Alice knows it is part of this protocol exchange
  - Cathy generated  $k_s$  because only she, Alice know  $k_A$
  - Enciphered part belongs to exchange as  $r_1$  matches  $r_1$  in encrypted part of first message

# Argument: Bob talking to Alice

---

- Third message
  - If  $n$  matches second message, Bob knows it is part of this protocol exchange
  - Cathy generated  $k_s$  because only she, Bob know  $k_B$
  - Enciphered part belongs to exchange as  $r_2$  matches  $r_2$  in encrypted part of second message

# Replay Attack

---

- Eve acquires old  $k_s$ , message in third step
  - $n \parallel \{ r_1 \parallel k_s \} k_A \parallel \{ r_2 \parallel k_s \} k_B$
- Eve forwards appropriate part to Alice
  - Alice has no ongoing key exchange with Bob:  $n$  matches nothing, so is rejected
  - Alice has ongoing key exchange with Bob:  $n$  does not match, so is again rejected
    - If replay is for the current key exchange, *and* Eve sent the relevant part *before* Bob did, Eve could simply listen to traffic; no replay involved

# Kerberos

---

- Authentication system
  - Based on Needham-Schroeder with Denning-Sacco modification
  - Central server plays role of trusted third party (“Cathy”)
- Ticket
  - Issuer vouches for identity of requester of service
- Authenticator
  - Identifies sender

# Idea

---

- User  $u$  authenticates to Kerberos server
  - Obtains ticket  $T_{u,TGS}$  for ticket granting service (TGS)
- User  $u$  wants to use service  $s$ :
  - User sends authenticator  $A_u$ , ticket  $T_{u,TGS}$  to TGS asking for ticket for service
  - TGS sends ticket  $T_{u,s}$  to user
  - User sends  $A_u$ ,  $T_{u,s}$  to server as request to use  $s$
- Details follow

# Ticket

---

- Credential saying issuer has identified ticket requester
- Example ticket issued to user  $u$  for service  $s$

$$T_{u,s} = s \parallel \{ u \parallel u\text{'s address} \parallel \text{valid time} \parallel k_{u,s} \} k_s$$

where:

- $k_{u,s}$  is session key for user and service
- Valid time is interval for which ticket valid
- $u$ 's address may be IP address or something else
  - Note: more fields, but not relevant here

# Authenticator

---

- Credential containing identity of sender of ticket
  - Used to confirm sender is entity to which ticket was issued
- Example: authenticator user  $u$  generates for service  $s$

$$A_{u,s} = \{ u \parallel \text{generation time} \parallel k_t \} k_{u,s}$$

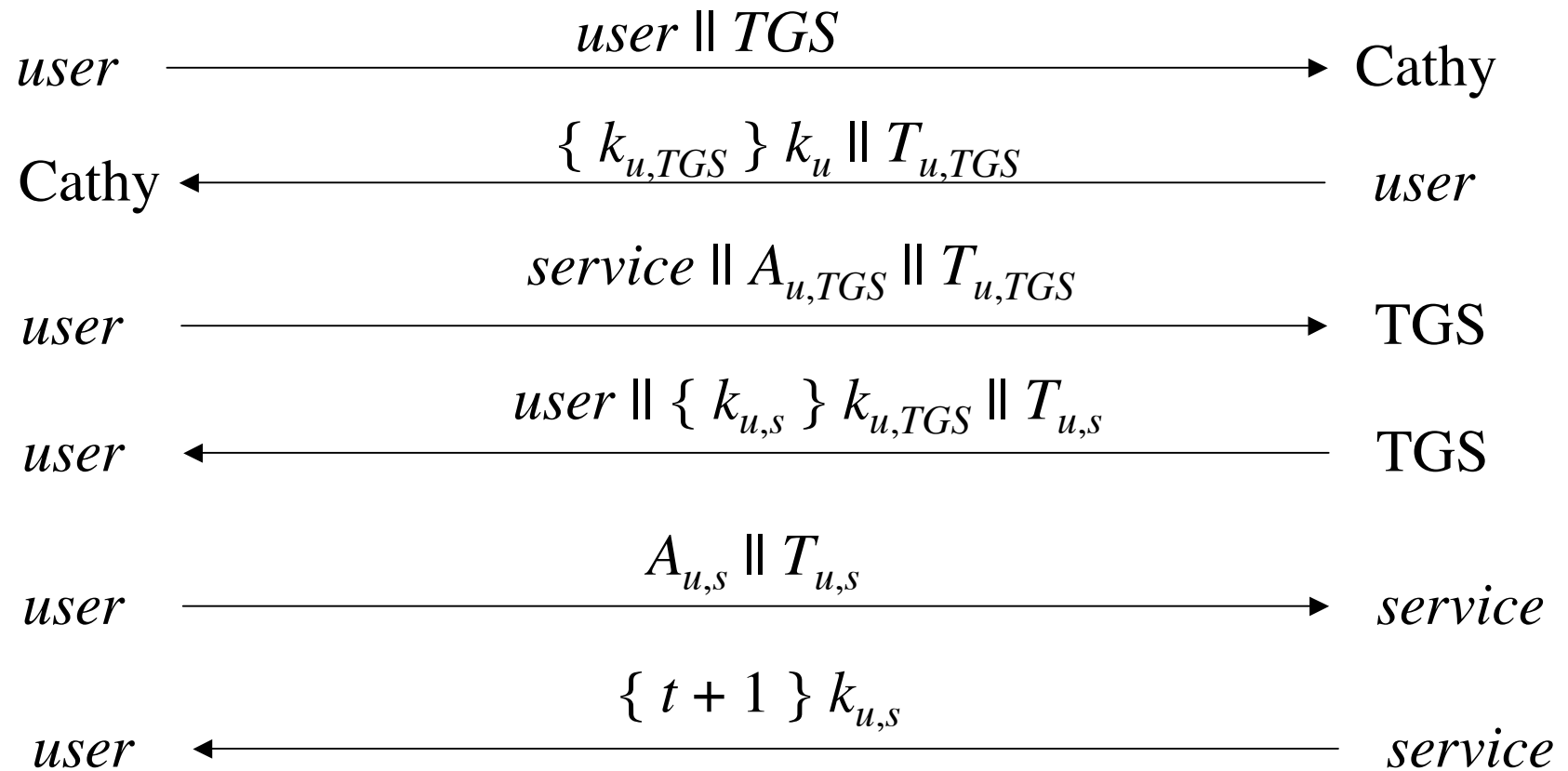
where:

- $k_t$  is alternate session key
- Generation time is when authenticator generated
  - Note: more fields, not relevant here



# Protocol

---



# Analysis

---

- First two steps get user ticket to use TGS
  - User  $u$  can obtain session key only if  $u$  knows key shared with Cathy
- Next four steps show how  $u$  gets and uses ticket for service  $s$ 
  - Service  $s$  validates request by checking sender (using  $A_{u,s}$ ) is same as entity ticket issued to
  - Step 6 optional; used when  $u$  requests confirmation

# Problems

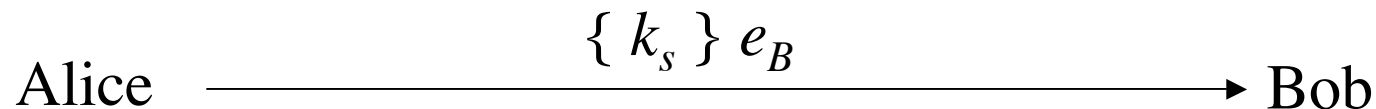
---

- Relies on synchronized clocks
  - If not synchronized and old tickets, authenticators not cached, replay is possible
- Tickets have some fixed fields
  - Dictionary attacks possible
  - Kerberos 4 session keys weak (had much less than 56 bits of randomness); researchers at Purdue found them from tickets in minutes

# Public Key Key Exchange

---

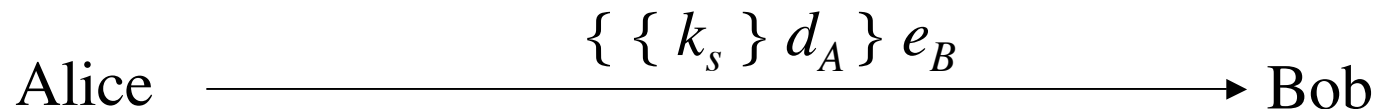
- Here interchange keys known
  - $e_A, e_B$  Alice and Bob's public keys known to all
  - $d_A, d_B$  Alice and Bob's private keys known only to owner
- Simple protocol
  - $k_s$  is desired session key



# Problem and Solution

---

- Vulnerable to forgery or replay
  - Because  $e_B$  known to anyone, Bob has no assurance that Alice sent message
- Simple fix uses Alice's private key
  - $k_s$  is desired session key

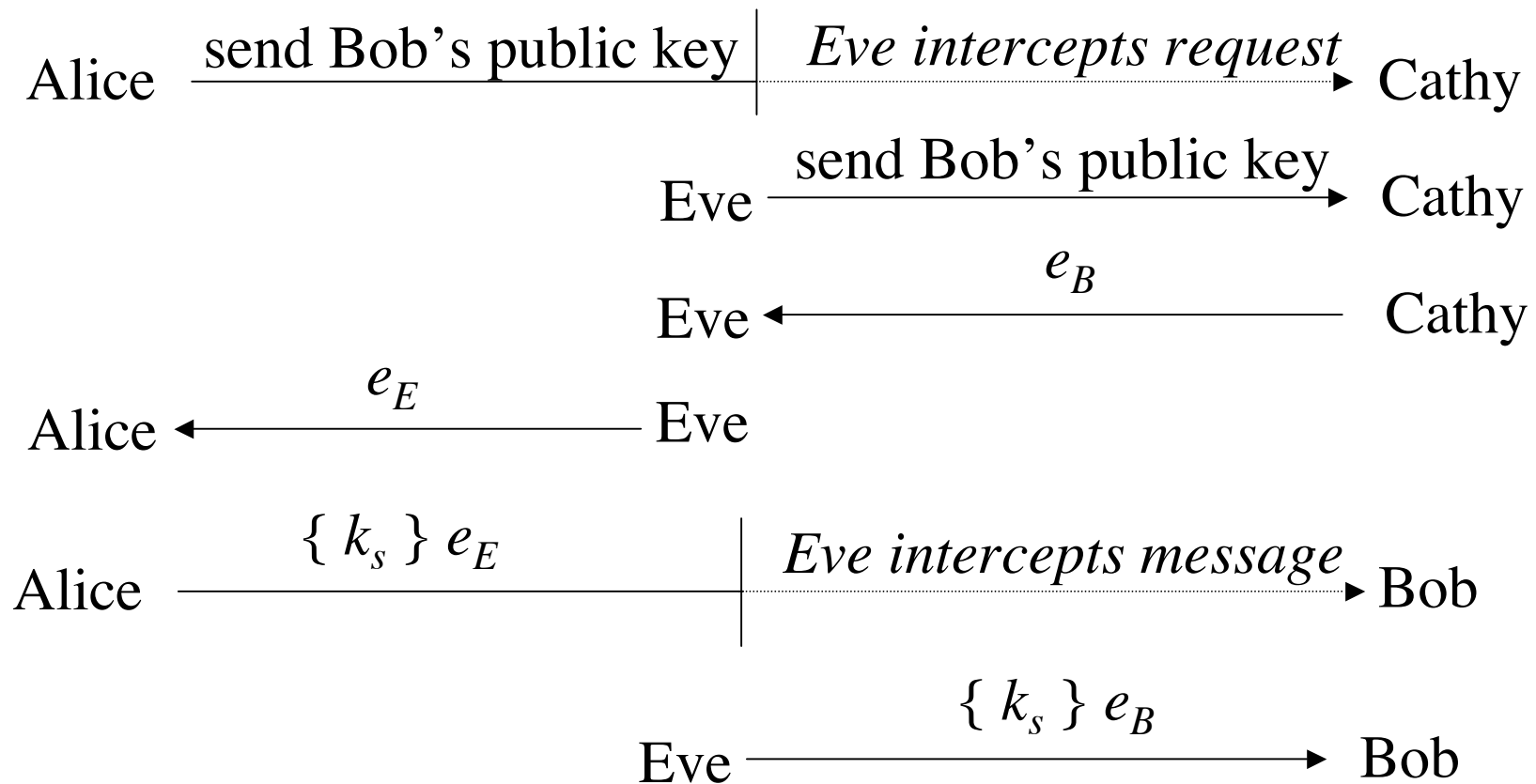


# Notes

---

- Can include message enciphered with  $k_s$
- Assumes Bob has Alice's public key, and *vice versa*
  - If not, each must get it from public server
  - If keys not bound to identity of owner, attacker Eve can launch a *man-in-the-middle* attack (next slide; Cathy is public server providing public keys)
    - Solution to this (binding identity to keys) discussed later as public key infrastructure (PKI)

# Man-in-the-Middle Attack



# Key Generation

---

- Goal: generate keys that are difficult to guess
- Problem statement: given a set of  $K$  potential keys, choose one randomly
  - Equivalent to selecting a random number between 0 and  $K-1$  inclusive
- Why is this hard: generating random numbers
  - Actually, numbers are usually *pseudo-random*, that is, generated by an algorithm



# What is “Random”?

---

- *Sequence of cryptographically random numbers:* a sequence of numbers  $n_1, n_2, \dots$  such that for any integer  $k > 0$ , an observer cannot predict  $n_k$  even if all of  $n_1, \dots, n_{k-1}$  are known
  - Best: physical source of randomness
    - Random pulses
    - Electromagnetic phenomena
    - Characteristics of computing environment such as disk latency
    - Ambient background noise

# What is “Pseudorandom”?

---

- *Sequence of cryptographically pseudorandom numbers*: sequence of numbers intended to simulate a sequence of cryptographically random numbers but generated by an algorithm
  - Very difficult to do this well
    - Linear congruential generators [ $n_k = (an_{k-1} + b) \bmod n$ ] broken
    - Polynomial congruential generators [ $n_k = (a_j n_{k-1}^j + \dots + a_1 n_{k-1} + a_0) \bmod n$ ] broken too
    - Here, “broken” means next number in sequence can be determined

# Best Pseudorandom Numbers

---

- *Strong mixing function*: function of 2 or more inputs with each bit of output depending on some nonlinear function of all input bits
  - Examples: DES, MD5, SHA-1
  - Use on UNIX-based systems:  

```
(date; ps gaux) | md5
```

where “ps gaux” lists all information about all processes on system

# Cryptographic Key Infrastructure

---

- Goal: bind identity to key
- Classical: not possible as all keys are shared
  - Use protocols to agree on a shared key (see earlier)
- Public key: bind identity to public key
  - Crucial as people will use key to communicate with principal whose identity is bound to key
  - Erroneous binding means no secrecy between principals
  - Assume principal identified by an acceptable name

# Certificates

---

- Create token (message) containing
  - Identity of principal (here, Alice)
  - Corresponding public key
  - Timestamp (when issued)
  - Other information (perhaps identity of signer)signed by trusted authority (here, Cathy)

$$C_A = \{ e_A \parallel \text{Alice} \parallel T \} d_C$$

# Use

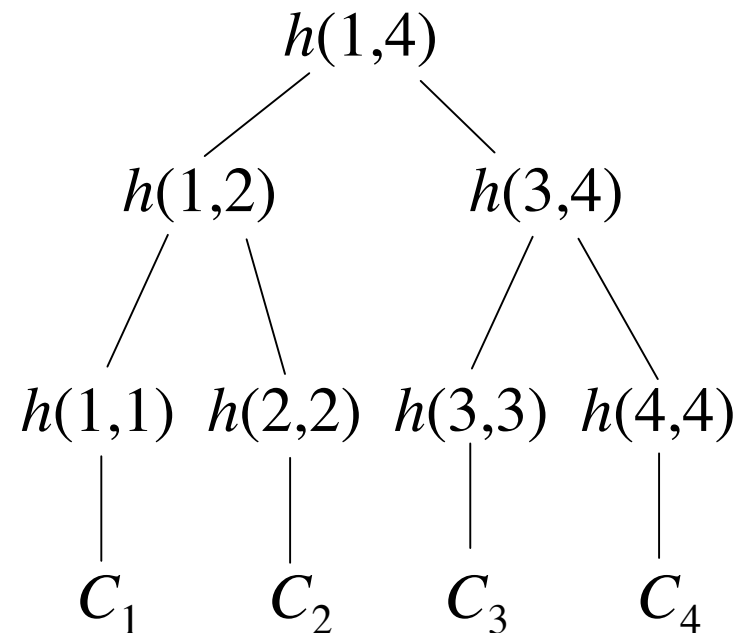
---

- Bob gets Alice's certificate
  - If he knows Cathy's public key, he can decipher the certificate
    - When was certificate issued?
    - Is the principal Alice?
  - Now Bob has Alice's public key
- Problem: Bob needs Cathy's public key to validate certificate
  - Problem pushed "up" a level
  - Two approaches: Merkle's tree, signature chains

# Merkle's Tree Scheme

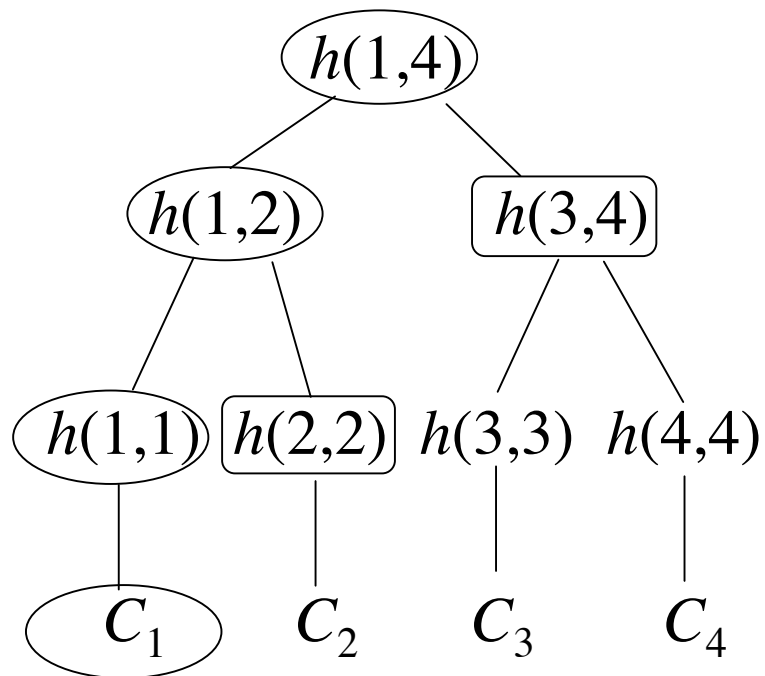
---

- Keep certificates in a file
  - Changing any certificate changes the file
  - Use crypto hash functions to detect this
- Define hashes recursively
  - $h$  is hash function
  - $C_i$  is certificate  $i$
- Hash of file ( $h(1,4)$  in example) known to all



# Validation

---



- To validate  $C_1$ :
  - Compute  $h(1, 1)$
  - Obtain  $h(2, 2)$
  - Compute  $h(1, 2)$
  - Obtain  $h(3, 4)$
  - Compute  $h(1, 4)$
  - Compare to known  $h(1, 4)$
- Need to know hashes of children of nodes on path that are not computed



# Details

---

- $f: D \times D \rightarrow D$  maps bit strings to bit strings
- $h: N \times N \rightarrow D$  maps integers to bit strings
  - if  $i \geq j$ ,  $h(i, j) = f(C_i, C_j)$
  - if  $i < j$ ,  
 $h(i, j) = f(h(i, \lfloor (i+j)/2 \rfloor), h(\lfloor (i+j)/2 \rfloor + 1, j))$

# Problem

---

- File must be available for validation
  - Otherwise, can't recompute hash at root of tree
  - Intermediate hashes would do
- Not practical in most circumstances
  - Too many certificates and users
  - Users and certificates distributed over widely separated systems

# Certificate Signature Chains

---

- Create certificate
  - Generate hash of certificate
  - Encipher hash with issuer's private key
- Validate
  - Obtain issuer's public key
  - Decipher enciphered hash
  - Recompute hash from certificate and compare
- Problem: getting issuer's public key

# X.509 Chains

---

- Some certificate components in X.509v3:
  - Version
  - Serial number
  - Signature algorithm identifier: hash algorithm
  - Issuer's name; uniquely identifies issuer
  - Interval of validity
  - Subject's name; uniquely identifies subject
  - Subject's public key
  - Signature: enciphered hash

# X.509 Certificate Validation

---

- Obtain issuer's public key
  - The one for the particular signature algorithm
- Decipher signature
  - Gives hash of certificate
- Recompute hash from certificate and compare
  - If they differ, there's a problem
- Check interval of validity
  - This confirms that certificate is current

# Issuers

---

- *Certification Authority (CA)*: entity that issues certificates
  - Multiple issuers pose validation problem
  - Alice's CA is Cathy; Bob's CA is Don; how can Alice validate Bob's certificate?
  - Have Cathy and Don cross-certify
    - Each issues certificate for the other

# Validation and Cross-Certifying

---

- Certificates:
  - Cathy<<Alice>>
  - Dan<<Bob>
  - Cathy<<Dan>>
  - Dan<<Cathy>>
- Alice validates Bob's certificate
  - Alice obtains Cathy<<Dan>>
  - Alice uses (known) public key of Cathy to validate Cathy<<Dan>>
  - Alice uses Cathy<<Dan>> to validate Dan<<Bob>>

# PGP Chains

---

- OpenPGP certificates structured into packets
  - One public key packet
  - Zero or more signature packets
- Public key packet:
  - Version (3 or 4; 3 compatible with all versions of PGP, 4 not compatible with older versions of PGP)
  - Creation time
  - Validity period (not present in version 3)
  - Public key algorithm, associated parameters
  - Public key



# OpenPGP Signature Packet

---

- Version 3 signature packet
  - Version (3)
  - Signature type (level of trust)
  - Creation time (when next fields hashed)
  - Signer's key identifier (identifies key to encipher hash)
  - Public key algorithm (used to encipher hash)
  - Hash algorithm
  - Part of signed hash (used for quick check)
  - Signature (enciphered hash)
- Version 4 packet more complex

# Signing

---

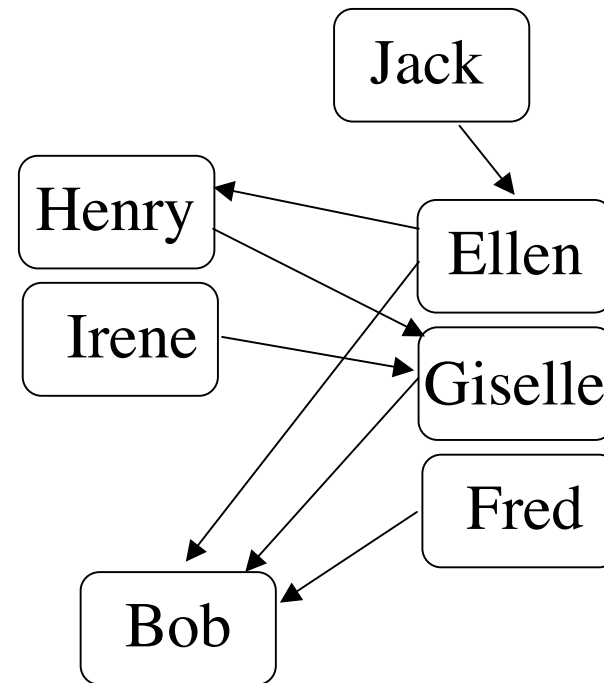
- Single certificate may have multiple signatures
- Notion of “trust” embedded in each signature
  - Range from “untrusted” to “ultimate trust”
  - Signer defines meaning of trust level (no standards!)
- All version 4 keys signed by subject
  - Called “self-signing”

# Validating Certificates

---

- Alice needs to validate Bob's OpenPGP cert
  - Does not know Fred, Giselle, or Ellen
- Alice gets Giselle's cert
  - Knows Henry slightly, but his signature is at "casual" level of trust
- Alice gets Ellen's cert
  - Knows Jack, so uses his cert to validate Ellen's, then hers to validate Bob's

Arrows show signatures  
Self signatures not shown



# Storing Keys

---

- Multi-user or networked systems: attackers may defeat access control mechanisms
  - Encipher file containing key
    - Attacker can monitor keystrokes to decipher files
    - Key will be resident in memory that attacker may be able to read
  - Use physical devices like “smart card”
    - Key never enters system
    - Card can be stolen, so have 2 devices combine bits to make single key

# Key Escrow

---

- *Key escrow system* allows authorized third party to recover key
  - Useful when keys belong to roles, such as system operator, rather than individuals
  - Business: recovery of backup keys
  - Law enforcement: recovery of keys that authorized parties require access to
- Goal: provide this without weakening cryptosystem
- Very controversial

# Desirable Properties

---

- Escrow system should not depend on encipherment algorithm
- Privacy protection mechanisms must work from end to end and be part of user interface
- Requirements must map to key exchange protocol
- System supporting key escrow must require all parties to authenticate themselves
- If message to be observable for limited time, key escrow system must ensure keys valid for that period of time only

# Components

---

- User security component
  - Does the encipherment, decipherment
  - Supports the key escrow component
- Key escrow component
  - Manages storage, use of data recovery keys
- Data recovery component
  - Does key recovery

# Example: ESS, Clipper Chip

---

- Escrow Encryption Standard
  - Set of interlocking components
  - Designed to balance need for law enforcement access to enciphered traffic with citizens' right to privacy
- Clipper chip prepares per-message escrow information
  - Each chip numbered uniquely by UID
  - Special facility programs chip
- Key Escrow Decrypt Processor (KEDP)
  - Available to agencies authorized to read messages



# User Security Component

---

- Unique device key  $k_{unique}$
- Non-unique family key  $k_{family}$
- Cipher is Skipjack
  - Classical cipher: 80 bit key, 64 bit input, output blocks
- Generates Law Enforcement Access Field (LEAF) of 128 bits:
  - $\{ UID \parallel \{ k_{session} \} k_{unique} \parallel hash \} k_{family}$
  - *hash*: 16 bit authenticator from session key and initialization vector

# Programming User Components

---

- Done in a secure facility
- Two escrow agencies needed
  - Agents from each present
  - Each supplies a random seed and key number
  - Family key components combined to get  $k_{family}$
  - Key numbers combined to make key component enciphering key  $k_{comp}$
  - Random seeds mixed with other data to produce sequence of unique keys  $k_{unique}$
- Each chip imprinted with UID,  $k_{unique}$ ,  $k_{family}$

# The Escrow Components

---

- During initialization of user security component, process creates  $k_{u1}$  and  $k_{u2}$  where  $k_{unique} = k_{u1} \oplus k_{u2}$ 
  - First escrow agency gets  $\{ k_{u1} \} k_{comp}$
  - Second escrow agency gets  $\{ k_{u2} \} k_{comp}$

# Obtaining Access

---

- Alice obtains legal authorization to read message
- She runs message LEAF through KEDP
  - LEAF is  $\{ \text{UID} \parallel \{ k_{\text{session}} \} k_{\text{unique}} \parallel \text{hash} \} k_{\text{family}}$
- KEDP uses (known)  $k_{\text{family}}$  to validate LEAF, obtain sending device's UID
- Authorization, LEAF taken to escrow agencies

# Agencies' Role

---

- Each validates authorization
- Each supplies  $\{ k_{ui} \} k_{comp}$ , corresponding key number
- KEDP takes these and LEAF:
  - Key numbers produce  $k_{comp}$
  - $k_{comp}$  produces  $k_{u1}$  and  $k_{u2}$
  - $k_{u1}$  and  $k_{u2}$  produce  $k_{unique}$
  - $k_{unique}$  and LEAF produce  $k_{session}$

# Problems

---

- *hash* too short
  - LEAF 128 bits, so given a hash:
    - $2^{112}$  LEAFs show this as a valid hash
    - 1 has actual session key, UID
    - Takes about 42 minutes to generate a LEAF with a valid hash but meaningless session key and UID
      - Turns out deployed devices would prevent this attack
  - Scheme does not meet temporal requirement
    - As  $k_{unique}$  fixed for each unit, once message is read, any future messages can be read

# Yaksha Security System

---

- Key escrow system meeting all 5 criteria
- Based on RSA, central server
  - Central server (Yaksha server) generates session key
- Each user has 2 private keys
  - Alice's modulus  $n_A$ , public key  $e_A$
  - First private key  $d_{AA}$  known only to Alice
  - Second private key  $d_{AY}$  known only to Yaksha central server
  - $d_{AA} d_{AY} = d_A \pmod{\phi(n_A)}$

# Alice and Bob

---

- Alice wants to send message to Bob
  - Alice asks Yaksha server for session key
  - Yaksha server generates  $k_{session}$
  - Yaksha server sends Alice the key as:

$$C_A = (k_{session})^{d_{AyeA}} \bmod n_A$$

- Alice computes

$$(C_A)^{d_{AA}} \bmod n_A = k_{session}$$



# Analysis

---

- Authority can read only one message per escrowed key
  - Meets requirement 5 (temporal one), because “time” interpreted as “session”
- Independent of message enciphering key
  - Meets requirement 1
  - Interchange algorithm, keys fixed
- Others met by supporting infrastructure

# Alternate Approaches

---

- Tie to time
  - Session key not given as escrow key, but related key is
  - To derive session key, must solve instance of discrete log problem
- Tie to probability
  - Oblivious transfer: message received with specified probability
  - Idea: *translucent cryptography* allows fraction  $f$  of messages to be read by third party
  - Not key escrow, but similar in spirit

# Key Revocation

---

- Certificates invalidated *before* expiration
  - Usually due to compromised key
  - May be due to change in circumstance (*e.g.*, someone leaving company)
- Problems
  - Entity revoking certificate authorized to do so
  - Revocation information circulates to everyone fast enough
    - Network delays, infrastructure problems may delay information

# CRLs

---

- *Certificate revocation list* lists certificates that are revoked
- X.509: only certificate issuer can revoke certificate
  - Added to CRL
- PGP: signers can revoke signatures; owners can revoke certificates, or allow others to do so
  - Revocation message placed in PGP packet and signed
  - Flag marks it as revocation message

# Digital Signature

---

- Construct that authenticated origin, contents of message in a manner provable to a disinterested third party (“judge”)
- Sender cannot deny having sent message (service is “nonrepudiation”)
  - Limited to *technical* proofs
    - Inability to deny one’s cryptographic key was used to sign
  - One could claim the cryptographic key was stolen or compromised
    - Legal proofs, *etc.*, probably required; not dealt with here

# Common Error

---

- Classical: Alice, Bob share key  $k$ 
  - Alice sends  $m \parallel \{ m \}_k$  to Bob

This is a digital signature

**WRONG**

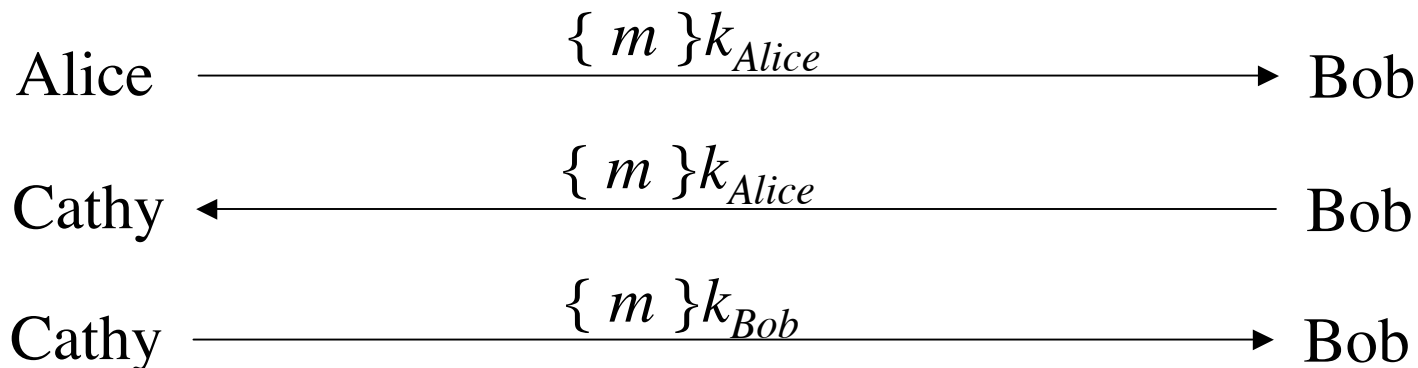
**This is not a digital signature**

- Why? Third party cannot determine whether Alice or Bob generated message

# Classical Digital Signatures

---

- Require trusted third party
  - Alice, Bob each share keys with trusted party Cathy
- To resolve dispute, judge gets  $\{ m \} k_{Alice}$ ,  $\{ m \} k_{Bob}$ , and has Cathy decipher them; if messages matched, contract was signed



# Public Key Digital Signatures

---

- Alice's keys are  $d_{Alice}, e_{Alice}$

- Alice sends Bob

$$m \parallel \{ m \}_{d_{Alice}}$$

- In case of dispute, judge computes

$$\{ \{ m \}_{d_{Alice}} \}_{e_{Alice}}$$

- and if it is  $m$ , Alice signed message
  - She's the only one who knows  $d_{Alice}$ !



# RSA Digital Signatures

---

- Use private key to encipher message
  - Protocol for use is *critical*
- Key points:
  - Never sign random documents, and when signing, always sign hash and never document
    - Mathematical properties can be turned against signer
  - Sign message first, then encipher
    - Changing public keys causes forgery

# Attack #1

---

- Example: Alice, Bob communicating
  - $n_A = 95, e_A = 59, d_A = 11$
  - $n_B = 77, e_B = 53, d_B = 17$
- 26 contracts, numbered 00 to 25
  - Alice has Bob sign 05 and 17:
    - $c = m^{d_B} \bmod n_B = 05^{17} \bmod 77 = 3$
    - $c = m^{d_B} \bmod n_B = 17^{17} \bmod 77 = 19$
  - Alice computes  $05 \times 17 \bmod 77 = 08$ ; corresponding signature is  $03 \times 19 \bmod 77 = 57$ ; claims Bob signed 08
  - Judge computes  $c^{e_B} \bmod n_B = 57^{53} \bmod 77 = 08$ 
    - Signature validated; Bob is toast

# Attack #2: Bob's Revenge

---

- Bob, Alice agree to sign contract 06
- Alice enciphers, then signs:  
$$(m^{e_B} \bmod 77)^{d_A} \bmod n_A = (06^{53} \bmod 77)^{11} \bmod 95 = 63$$
- Bob now changes his public key
  - Computes  $r$  such that  $13^r \bmod 77 = 6$ ; say,  $r = 59$
  - Computes  $re_B \bmod \phi(n_B) = 59 \times 53 \bmod 60 = 7$
  - Replace public key  $e_B$  with 7, private key  $d_B = 43$
- Bob claims contract was 13. Judge computes:
  - $(63^{59} \bmod 95)^{43} \bmod 77 = 13$
  - Verified; now Alice is toast

# El Gamal Digital Signature

---

- Relies on discrete log problem
- Choose  $p$  prime,  $g, d < p$ ; compute  $y = g^d \bmod p$
- Public key:  $(y, g, p)$ ; private key:  $d$
- To sign contract  $m$ :
  - Choose  $k$  relatively prime to  $p-1$ , and not yet used
  - Compute  $a = g^k \bmod p$
  - Find  $b$  such that  $m = (da + kb) \bmod p-1$
  - Signature is  $(a, b)$
- To validate, check that
  - $y^a a^b \bmod p = g^m \bmod p$

# Example

---

- Alice chooses  $p = 29$ ,  $g = 3$ ,  $d = 6$   
 $y = 3^6 \bmod 29 = 4$
- Alice wants to send Bob signed contract 23
  - Chooses  $k = 5$  (relatively prime to 28)
  - This gives  $a = g^k \bmod p = 3^5 \bmod 29 = 11$
  - Then solving  $23 = (6 \times 11 + 5b) \bmod 28$  gives  $b = 25$
  - Alice sends message 23 and signature (11, 25)
- Bob verifies signature:  $g^m \bmod p = 3^{23} \bmod 29 = 8$  and  $y^a a^b \bmod p = 4^{11} 11^{25} \bmod 29 = 8$ 
  - They match, so Alice signed

# Attack

---

- Eve learns  $k$ , corresponding message  $m$ , and signature  $(a, b)$ 
  - Extended Euclidean Algorithm gives  $d$ , the private key
- Example from above: Eve learned Alice signed last message with  $k = 5$ 
$$m = (da + kb) \bmod p-1 = (11d + 5 \times 25) \bmod 28$$
so Alice's private key is  $d = 6$

# Key Points

---

- Key management critical to effective use of cryptosystems
  - Different levels of keys (session vs. interchange)
- Keys need infrastructure to identify holders, allow revoking
  - Key escrowing complicates infrastructure
- Digital signatures provide integrity of origin and content
  - Much easier with public key cryptosystems than with classical cryptosystems