Puzzle of the Day

A *lattice* is a mathematical structure composed of a set S and a relation r. If x, y, and z are elements of S, and r is a relation over those elements, then (S, r) is a lattice if and only if the following conditions are met:

1. r is reflexive, so for every element x in S, xrx.
2. r is antisymmetric, so if xry and yrx, then $x = y$.
3. r is transitive, so if xry and yrz, then xrz.
4. Every pair of elements in S has a lower bound. In other words, there is a z in S such that zrx and zry for every pair (x, y).
5. Every pair of elements in S has an upper bound. In other words, there is a z in S such that xrz and yrz for every pair (x, y).

Does $(\{ 1, 2, 3, 4, 5 \}, \leq)$ form a lattice? How about ({true, false}, **and**)?