Basic Security Theorem

- Define action, secure formally
  - Using a bit of foreshadowing for “secure”
- Restate properties formally
  - Simple security condition
  - *-property
  - Discretionary security property
- State conditions for properties to hold
- State Basic Security Theorem

Action

- A request and decision that causes the system to move from one state to another
  - Final state may be the same as initial state
- $(r, d, v, v') \in R \times D \times V \times V$ is an *action* of $\Sigma(R, D, W, z_0)$ iff there is an $(x, y, z) \in \Sigma(R, D, W, z_0)$ and a $t \in N$ such that $(r, d, v, v') = (x_t, y_t, z_t, z_{t-1})$
  - Request r made when system in state v; decision d moves system into (possibly the same) state v'
  - Correspondence with (x_t, y_t, z_t, z_{t-1}) makes states, requests, part of a sequence

Simple Security Condition

- $(s, o, p) \in S \times O \times P$ satisfies the simple security condition relative to f (written *ssc rel f*) iff one of the following holds:
  1. $p = \underline{e}$ or $p = \underline{a}$
  2. $p = \underline{r}$ or $p = \underline{w}$ and $f_c(s)$ *dom* $f_o(o)$
- Holds vacuously if rights do not involve reading
- If all elements of b satisfy *ssc rel f*, then state satisfies simple security condition
- If all states satisfy simple security condition, system satisfies simple security condition

Necessary and Sufficient

- $\Sigma(R, D, W, z_0)$ satisfies the simple security condition for any secure state z_0 iff for every action $(r, d, (b, m, f, h), (b', m', f', h'))$, W satisfies
  - Every $(s, o, p) \in b - b'$ satisfies *ssc rel f*
  - Every $(s, o, p) \in b'$ that does not satisfy *ssc rel f* is not in b
- Note: “secure” means z_0 satisfies *ssc rel f*
- First says every (s, o, p) added satisfies *ssc rel f*; second says any (s, o, p) in b' that does not satisfy *ssc rel f* is deleted

*-Property

- $b(s\colon p_1, \ldots, p_n)$ set of all objects that s has $p_1, \ldots, p_n$ access to
- State (b, m, f, h) satisfies the *-property iff for each $s \in S$ the following hold:
    1. $b(s\colon \underline{a}) \neq \varnothing \Rightarrow [\forall o \in b(s\colon \underline{a})\ [\, f_o(o)\ dom\ f_c(s)\ ]\ ]$
    2. $b(s\colon \underline{w}) \neq \varnothing \Rightarrow [\forall o \in b(s\colon \underline{w})\ [\, f_o(o) = f_c(s)\ ]\ ]$
    3. $b(s\colon \underline{r}) \neq \varnothing \Rightarrow [\forall o \in b(s\colon \underline{r})\ [\, f_c(s)\ dom\ f_o(o)\ ]\ ]$
- Idea: for writing, object dominates subject; for reading, subject dominates object

*-Property

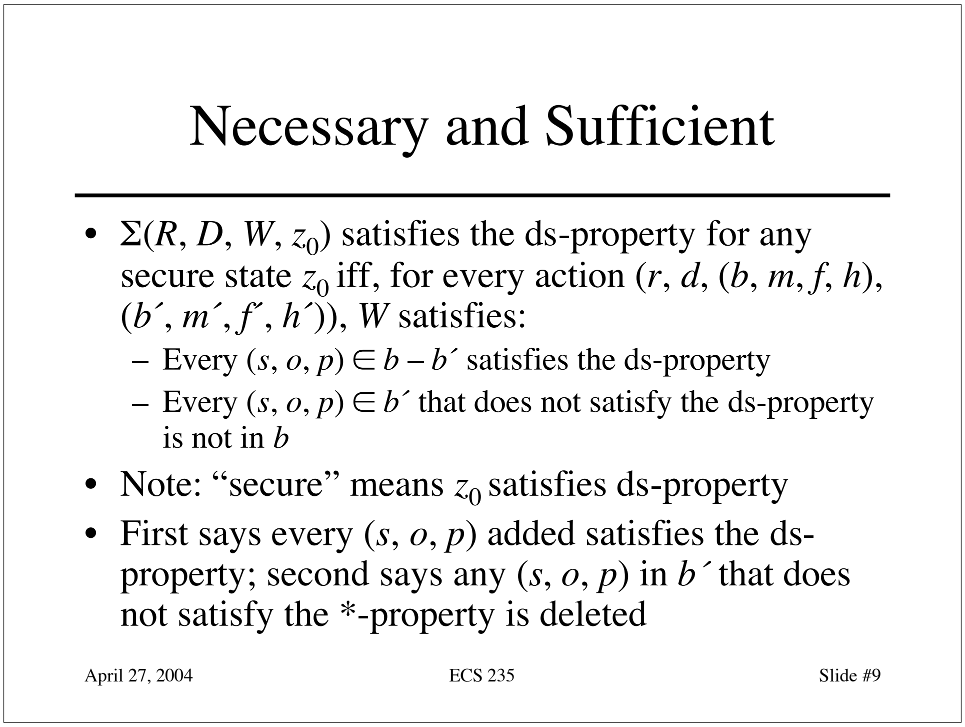
- If all states satisfy simple security condition, system satisfies simple security condition
- If a subset S' of subjects satisfy *-property, then *-property satisfied relative to $S' \subseteq S$
- Note: tempting to conclude that *-property includes simple security condition, but this is false
    - See condition placed on $\underline{\mathrm{w}}$ right for each

Necessary and Sufficient

- $\Sigma(R, D, W, z_0)$ satisfies the *-property relative to $S' \subseteq S$ for any secure state z_0 iff for every action $(r, d, (b, m, f, h), (b', m', f', h'))$, W satisfies the following for every $s \in S'$
  - Every $(s, o, p) \in b - b'$ satisfies the *-property relative to S'
  - Every $(s, o, p) \in b'$ that does not satisfy the *-property relative to S' is not in b
- Note: “secure” means z_0 satisfies *-property relative to S'
- First says every (s, o, p) added satisfies the *-property relative to S'; second says any (s, o, p) in b' that does not satisfy the *-property relative to S' is deleted

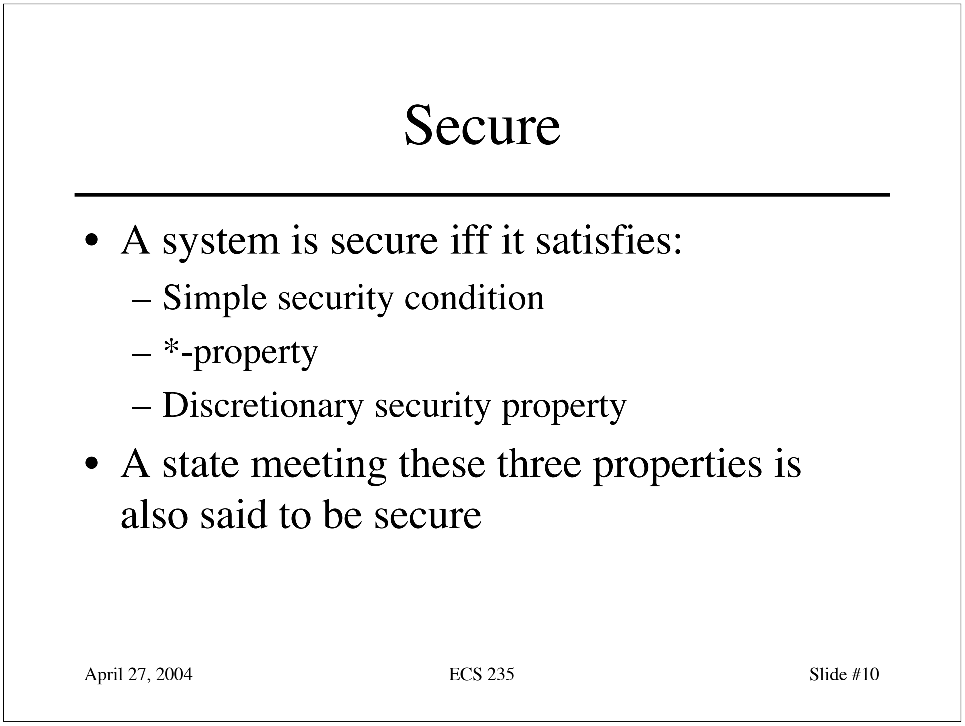
Discretionary Security Property

- State (b, m, f, h) satisfies the discretionary security property iff, for each $(s, o, p) \in b$, then $p \in m[s, o]$
- Idea: if s can read o, then it must have rights to do so in the access control matrix m
- This is the discretionary access control part of the model
  - The other two properties are the mandatory access control parts of the model



Necessary and Sufficient

- $\Sigma(R, D, W, z_0)$ satisfies the ds-property for any secure state z_0 iff, for every action $(r, d, (b, m, f, h), (b', m', f', h'))$, W satisfies:
  - Every $(s, o, p) \in b - b'$ satisfies the ds-property
  - Every $(s, o, p) \in b'$ that does not satisfy the ds-property is not in b
- Note: “secure” means z_0 satisfies ds-property
- First says every (s, o, p) added satisfies the ds-property; second says any (s, o, p) in b' that does not satisfy the *-property is deleted



Secure

- A system is secure iff it satisfies:
  - Simple security condition
  - *-property
  - Discretionary security property
- A state meeting these three properties is also said to be secure

Basic Security Theorem

- $\Sigma(R, D, W, z_0)$ is a secure system if z_0 is a secure state and W satisfies the conditions for the preceding three theorems
  - The theorems are on the slides titled "Necessary and Sufficient"

Rule

- $\rho: R \times V \rightarrow D \times V$
- Takes a state and a request, returns a decision and a (possibly new) state
- Rule ρ *ssc-preserving* if for all $(r, v) \in R \times V$ and v satisfying *ssc rel f*, $\rho(r, v) = (d, v')$ means that v' satisfies *ssc rel f′*.
  - Similar definitions for *-property, ds-property
  - If rule meets all 3 conditions, it is *security-preserving*

Unambiguous Rule Selection

- Problem: multiple rules may apply to a request in a state
  - if two rules act on a read request in state v …
- Solution: define relation $W(\omega)$ for a set of rules $\omega = \{ \rho_1, \ldots, \rho_m \}$ such that a state $(r, d, v, v') \in W(\omega)$ iff either
  - $d = \underline{i}$; or
  - for exactly one integer j, $\rho_j(r, v) = (d, v')$
- Either request is illegal, or only one rule applies

Rules Preserving *SSC*

- Let ω be set of *ssc*-preserving rules. Let state z_0 satisfy simple security condition. Then $\Sigma(R, D, W(\omega), z_0)$ satisfies simple security condition
  - Proof: by contradiction.
    - Choose $(x, y, z) \in \Sigma(R, D, W(\omega), z_0)$ as state not satisfying simple security condition; then choose $t \in N$ such that (x_t, y_t, z_t) is first appearance not meeting simple security condition
    - As $(x_t, y_t, z_t, z_{t-1}) \in W(\omega)$, there is unique rule $\rho \in \omega$ such that $\rho(x_t, z_{t-1}) = (y_t, z_t)$ and $y_t \neq \underline{i}$.
    - As ρ ssc-preserving, and z_{t-1} satisfies simple security condition, then z_t meets simple security condition, contradiction.

Adding States Preserving *SSC*

- Let $v = (b, m, f, h)$ satisfy simple security condition. Let $(s, o, p) \notin b$, $b' = b \cup \{ (s, o, p) \}$, and $v' = (b', m, f, h)$. Then v' satisfies simple security condition iff:
  1. Either $p = \underline{e}$ or $p = \underline{a}$; or
  2. Either $p = \underline{r}$ or $p = \underline{w}$, and $f_c(s)$ *dom* $f_o(o)$
  - Proof
    1. Immediate from definition of simple security condition and v' satisfying *ssc rel f*
    2. v' satisfies simple security condition means $f_c(s)$ *dom* $f_o(o)$, and for converse, $(s, o, p) \in b'$ satisfies *ssc rel f*, so v' satisfies simple security condition

Rules, States Preserving *-Property

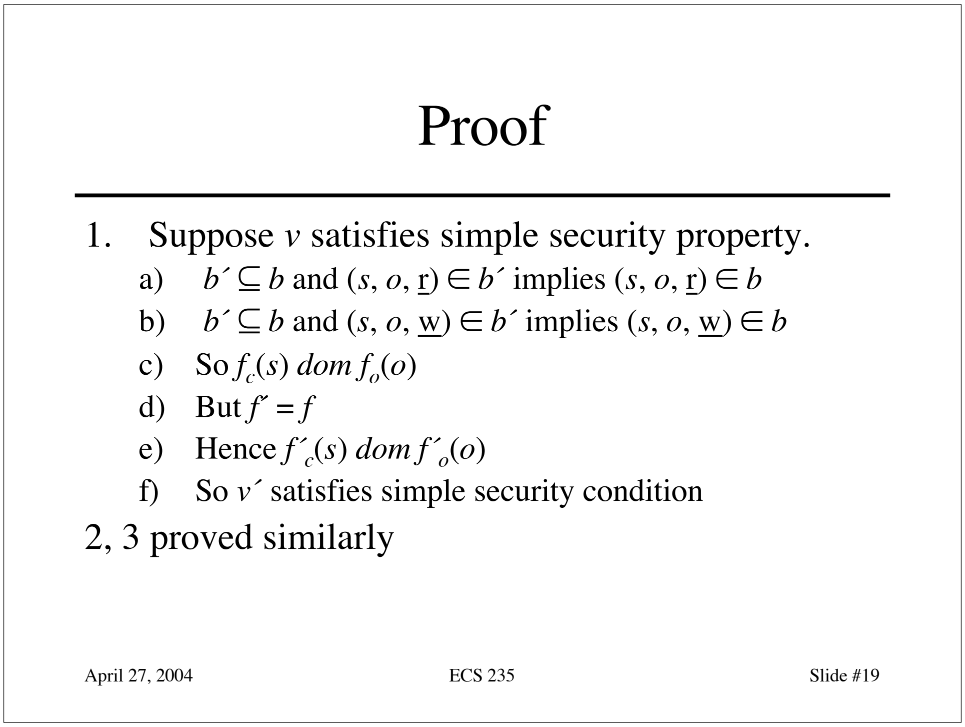
- Let ω be set of *-property-preserving rules, state z_0 satisfies *-property. Then $\Sigma(R, D, W(\omega), z_0)$ satisfies *-property
- Let $v = (b, m, f, h)$ satisfy *-property. Let $(s, o, p) \notin b$, $b' = b \cup \{ (s, o, p) \}$, and $v' = (b', m, f, h)$. Then v' satisfies *-property iff one of the following holds:
  1. $p = \underline{e}$ or $p = \underline{a}$
  2. $p = \underline{r}$ or $p = \underline{w}$ and $f_c(s)$ *dom* $f_o(o)$

Rules, States Preserving ds-Property

- Let ω be set of ds-property-preserving rules, state z_0 satisfies ds-property. Then $\Sigma(R, D, W(\omega), z_0)$ satisfies ds-property
- Let $v = (b, m, f, h)$ satisfy ds-property. Let $(s, o, p) \notin b$, $b' = b \cup \{ (s, o, p) \}$, and $v' = (b', m, f, h)$. Then v' satisfies ds-property iff $p \in m[s, o]$.

Combining

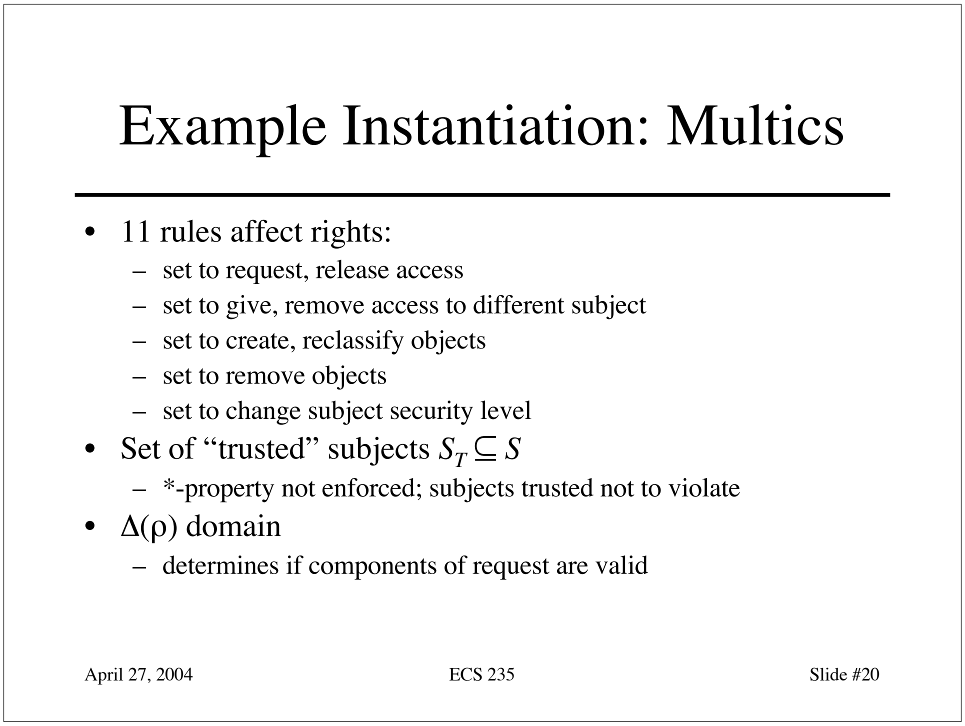
- Let ρ be a rule and $\rho(r, v) = (d, v')$, where $v = (b, m, f, h)$ and $v' = (b', m', f', h')$. Then:
  1. If $b' \subseteq b$, $f' = f$, and v satisfies the simple security condition, then v' satisfies the simple security condition
  2. If $b' \subseteq b$, $f' = f$, and v satisfies the *-property, then v' satisfies the *-property
  3. If $b' \subseteq b$, $m[s, o] \subseteq m'[s, o]$ for all $s \in S$ and $o \in O$, and v satisfies the ds-property, then v' satisfies the ds-property



Proof

1. Suppose v satisfies simple security property.
   a) $b' \subseteq b$ and $(s, o, \underline{r}) \in b'$ implies $(s, o, \underline{r}) \in b$
   b) $b' \subseteq b$ and $(s, o, \underline{w}) \in b'$ implies $(s, o, \underline{w}) \in b$
   c) So $f_c(s)$ *dom* $f_o(o)$
   d) But $f' = f$
   e) Hence $f'_c(s)$ *dom* $f'_o(o)$
   f) So v' satisfies simple security condition

2, 3 proved similarly



Example Instantiation: Multics

- 11 rules affect rights:
  - set to request, release access
  - set to give, remove access to different subject
  - set to create, reclassify objects
  - set to remove objects
  - set to change subject security level
- Set of “trusted” subjects $S_T \subseteq S$
  - *-property not enforced; subjects trusted not to violate
- $\Delta(\rho)$ domain
  - determines if components of request are valid

get-read Rule

- Request $r = (get, s, o, \underline{r})$
  - s gets (requests) the right to read o
- Rule is $\rho_1(r, v)$:

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**if** $(r \neq \Delta(\rho_1))$ **then** $\rho_1(r, v) = (\underline{i}, v)$;
**else if** $(f_s(s)\ dom\ f_o(o)$ **and** $[s \in S_T$ **or** $f_c(s)\ dom\ f_o(o)]$
    **and** $r \in m[s, o])$
            **then** $\rho_1(r, v) = (y, (b \cup \{ (s, o, \underline{r}) \}, m, f, h))$;
**else** $\rho_1(r, v) = (\underline{n}, v)$;
```

Security of Rule

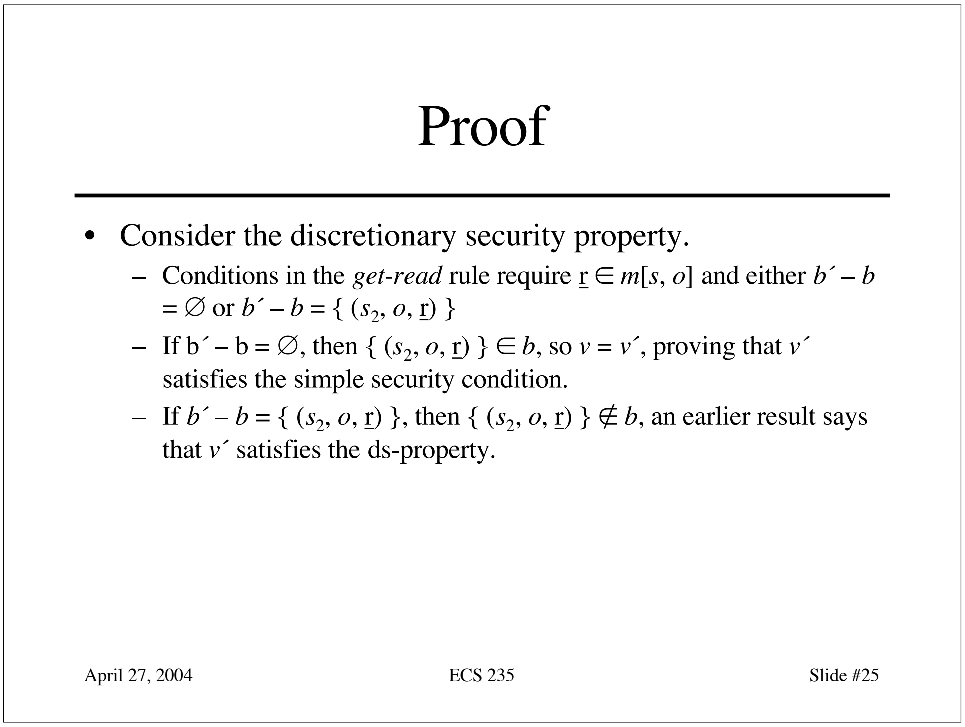
- The get-read rule preserves the simple security condition, the *-property, and the ds-property
  - Proof
    - Let v satisfy all conditions. Let $\rho_1(r, v) = (d, v')$. If $v' = v$ result is trivial. So let $v' = (b \cup \{ (s_2, o, \underline{r}) \}, m, f, h)$.

Proof

- Consider the simple security condition.
  - From the choice of v', either $b' - b = \varnothing\Delta$ or $b' - b = \{ (s_2, o, \underline{r}) \}$
  - If $b' - b = \varnothing$, then $\{ (s_2, o, \underline{r}) \} \in b$, so $v = v'$, proving that v' satisfies the simple security condition.
  - If $b' - b = \{ (s_2, o, \underline{r}) \}$, because the *get-read* rule requires that $f_c(s)$ *dom* $f_o(o)$, an earlier result says that v' satisfies the simple security condition.

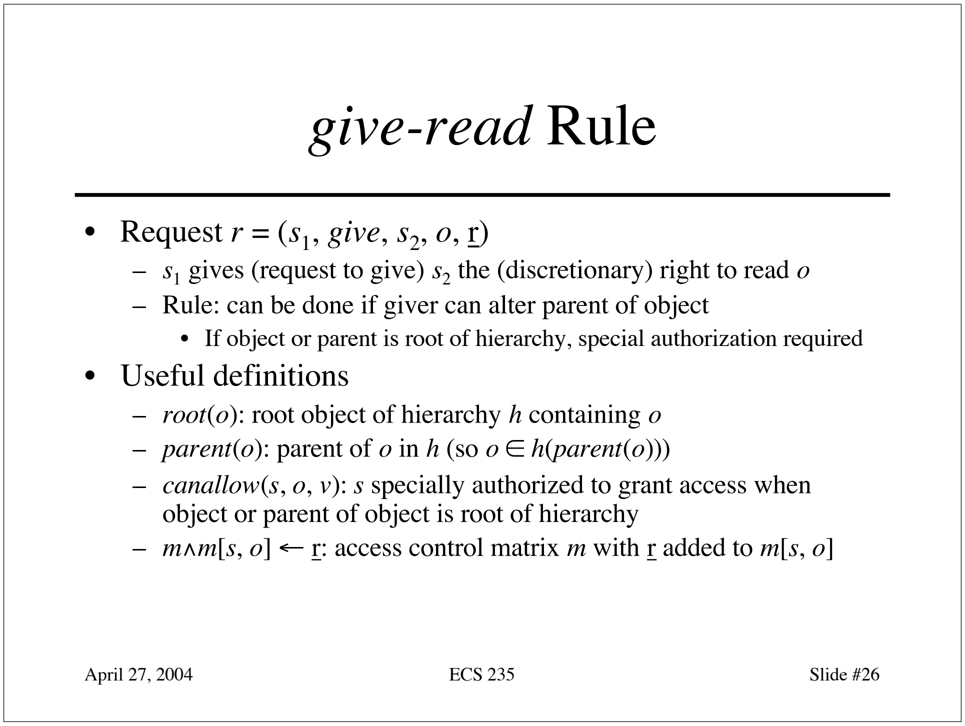
Proof

- Consider the *-property.
  - Either $s_2 \in S_T$ or $f_c(s)$ *dom* $f_o(o)$ from the definition of *get-read*
  - If $s_2 \in S_T$, then s_2 is trusted, so *-property holds by definition of trusted and S_T.
  - If $f_c(s)$ *dom* $f_o(o)$, an earlier result says that v' satisfies the simple security condition.



Proof

- Consider the discretionary security property.
  - Conditions in the *get-read* rule require $\underline{\mathrm{r}} \in m[s, o]$ and either $b' - b = \varnothing$ or $b' - b = \{ (s_2, o, \underline{\mathrm{r}}) \}$
  - If $\mathrm{b}' - \mathrm{b} = \varnothing$, then $\{ (s_2, o, \underline{\mathrm{r}}) \} \in b$, so $v = v'$, proving that v' satisfies the simple security condition.
  - If $b' - b = \{ (s_2, o, \underline{\mathrm{r}}) \}$, then $\{ (s_2, o, \underline{\mathrm{r}}) \} \notin b$, an earlier result says that v' satisfies the ds-property.



give-read Rule

- Request $r = (s_1, give, s_2, o, \underline{\mathrm{r}})$
  - s_1 gives (request to give) s_2 the (discretionary) right to read o
  - Rule: can be done if giver can alter parent of object
    - If object or parent is root of hierarchy, special authorization required
- Useful definitions
  - *root*(o): root object of hierarchy h containing o
  - *parent*(o): parent of o in h (so $o \in h(parent(o))$)
  - *canallow*(s, o, v): s specially authorized to grant access when object or parent of object is root of hierarchy
  - $m \wedge m[s, o] \leftarrow \underline{\mathrm{r}}$: access control matrix m with $\underline{\mathrm{r}}$ added to $m[s, o]$

give-read Rule

- Rule is $\rho_6(r, v)$:

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if ($r \neq \Delta(\rho_6)$) then $\rho_6(r, v) = (\underline{i}, v)$;
else if ([$o \neq root(o)$ and parent(o) $\neq$ root(o) and parent(o) $\in b(s_1{:}\underline{w})$] or
    [$parent(o) = root(o)$ and $canallow(s_1, o, v)$ ] or
    [$o = root(o)$ and $canallow(s_1, o, v)$ ])
            then $\rho_6(r, v) = (y, (b, m \wedge m[s_2, o] \leftarrow \underline{r}, f, h))$;
else $\rho_1(r, v) = (\underline{n}, v)$;
```
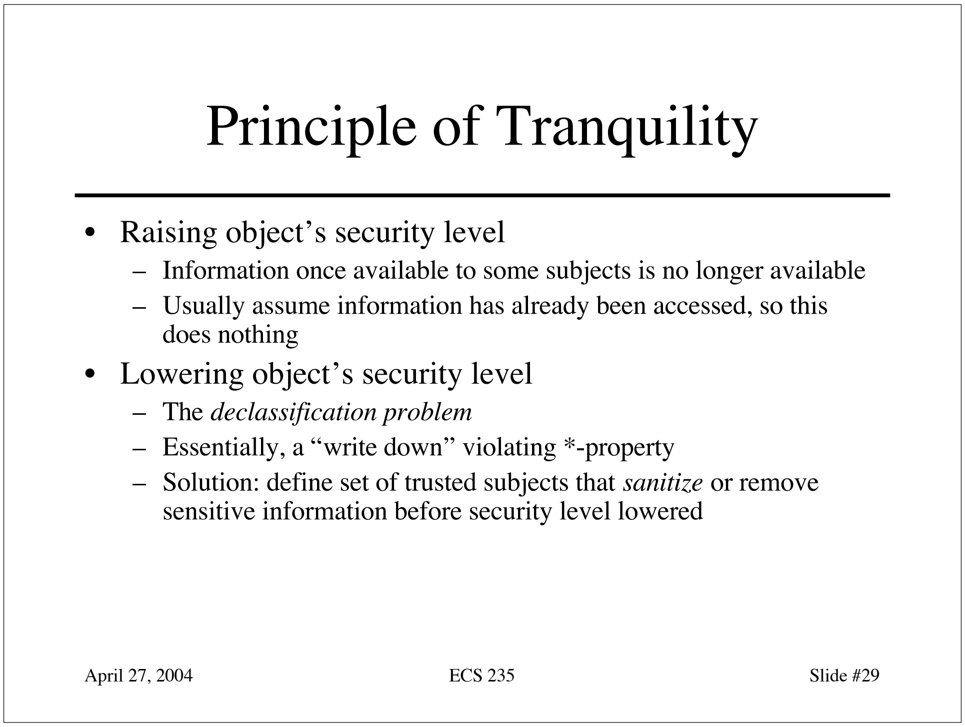
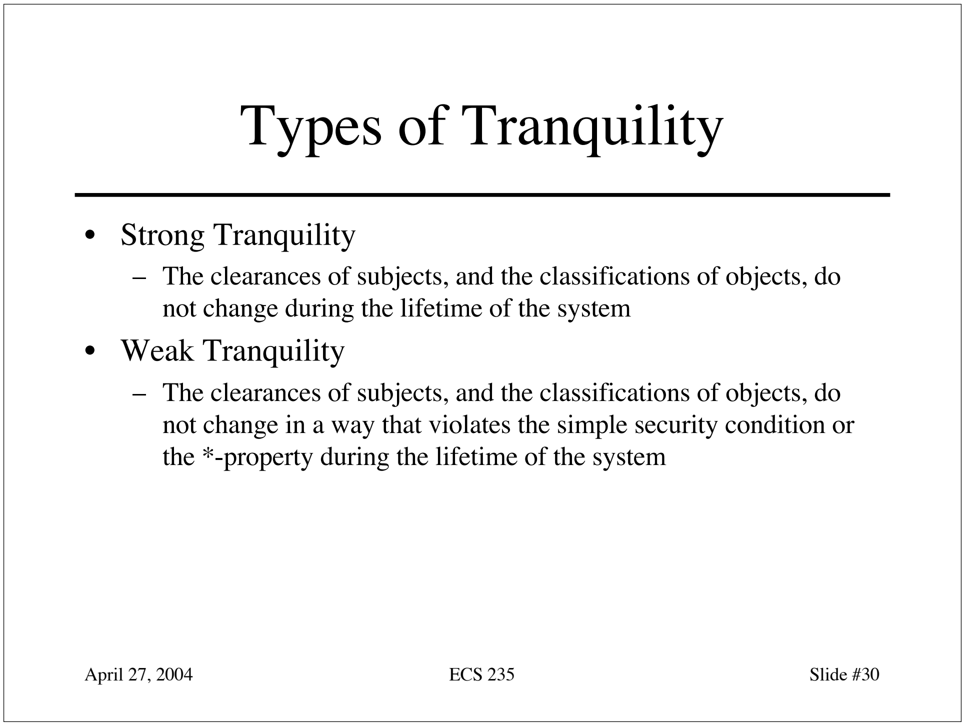

Security of Rule

- The give-read rule preserves the simple security condition, the *-property, and the ds-property
  - Proof: Let v satisfy all conditions. Let $\rho_1(r, v) = (d, v')$. If $v' = v$, result is trivial. So let $v' = (b, m[s_2, o] \leftarrow \underline{r}, f, h)$. $b' = b$, $f' = f$, $m[x, y] = m'[x, y]$ for all $x \in S$ and $y \in O$ such that $x \neq s$ and $y \neq o$, and $m[s, o] \subseteq m'[s, o]$. So, by earlier result, v' satisfies the simple security condition, the *-property, and the ds-property.



Principle of Tranquility

- Raising object’s security level
  - Information once available to some subjects is no longer available
  - Usually assume information has already been accessed, so this does nothing
- Lowering object’s security level
  - The *declassification problem*
  - Essentially, a “write down” violating *-property
  - Solution: define set of trusted subjects that *sanitize* or remove sensitive information before security level lowered



Types of Tranquility

- Strong Tranquility
  - The clearances of subjects, and the classifications of objects, do not change during the lifetime of the system
- Weak Tranquility
  - The clearances of subjects, and the classifications of objects, do not change in a way that violates the simple security condition or the *-property during the lifetime of the system

Example

- DG/UX System
  - Only a trusted user (security administrator) can lower object’s security level
  - In general, process MAC labels cannot change
    - If a user wants a new MAC label, needs to initiate new process
    - Cumbersome, so user can be designated as able to change process MAC label within a specified range

Controversy

- McLean:
  - “value of the BST is much overrated since there is a great deal more to security than it captures. Further, what is captured by the BST is so trivial that it is hard to imagine a realistic security model for which it does not hold.”
  - Basis: given assumptions known to be non-secure, BST can prove a non-secure system to be secure

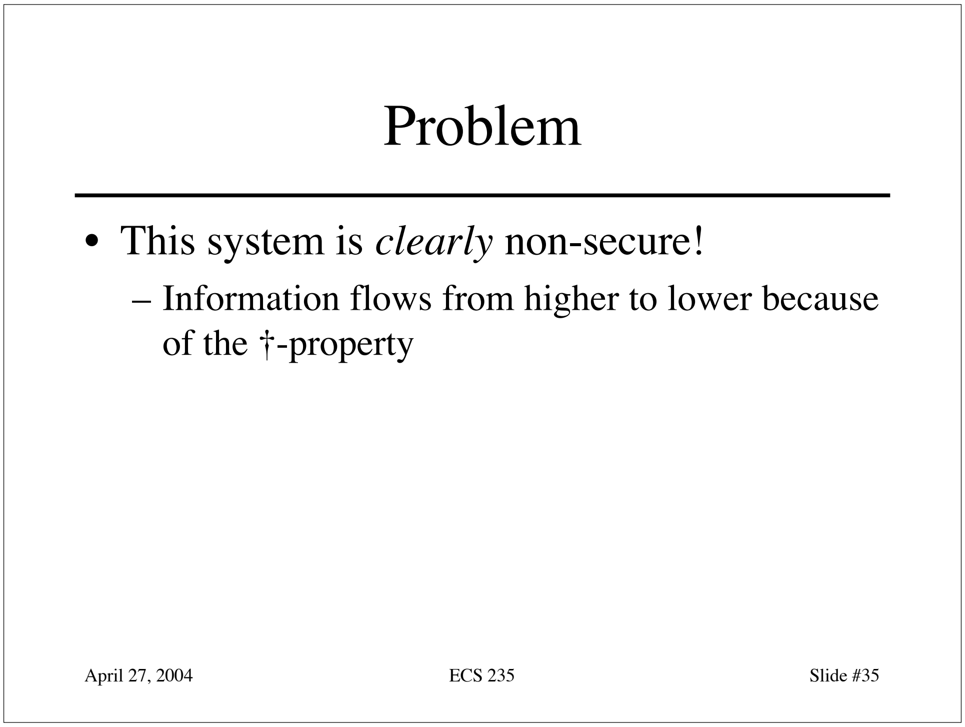
†-Property

- State (b, m, f, h) satisfies the †-property iff for each $s \in S$ the following hold:
  1. $b(s\text{: }\underline{\text{a}}) \neq \varnothing \Rightarrow [\forall o \in b(s\text{: }\underline{\text{a}})\ [\, f_c(s)\ dom\ f_o(o)\ ]\ ]$
  2. $b(s\text{: }\underline{\text{w}}) \neq \varnothing \Rightarrow [\forall o \in b(s\text{: }\underline{\text{w}})\ [\, f_o(o) = f_c(s)\ ]\ ]$
  3. $b(s\text{: }\underline{\text{r}}) \neq \varnothing \Rightarrow [\forall o \in b(s\text{: }\underline{\text{r}})\ [\, f_c(s)\ dom\ f_o(o)\ ]\ ]$
- Idea: for writing, subject dominates object; for reading, subject also dominates object
- Differs from *-property in that the mandatory condition for writing is reversed
  - For *-property, it's object dominates subject

Analogues

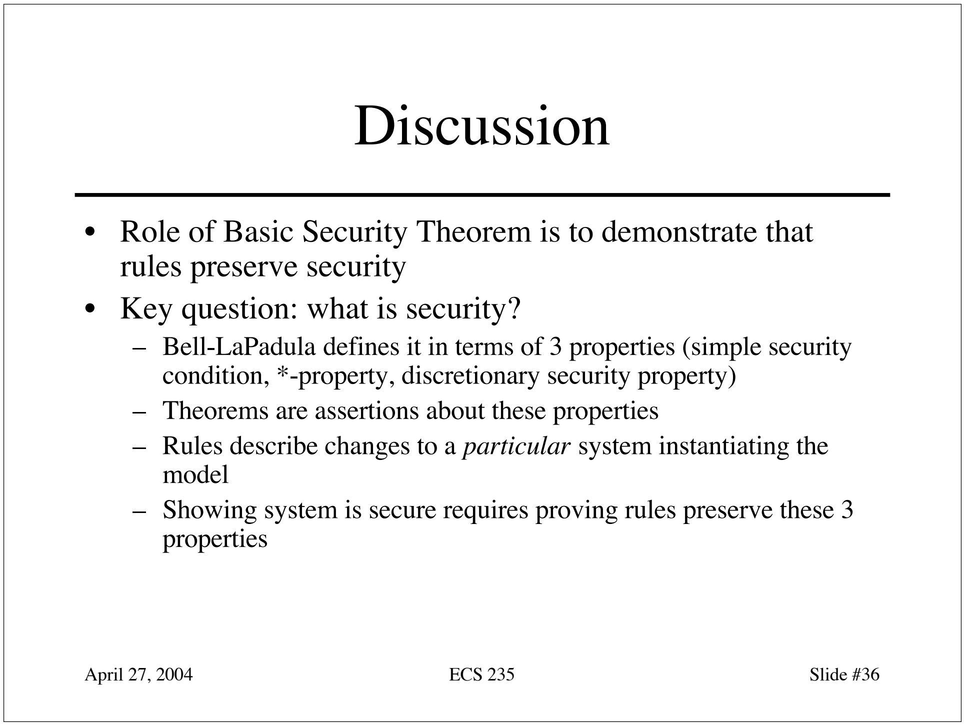
The following two theorems can be proved

- $\Sigma(R, D, W, z_0)$ satisfies the †-property relative to $S' \subseteq S$ for any secure state z_0 iff for every action $(r, d, (b, m, f, h), (b', m', f', h'))$, W satisfies the following for every $s \in S'$
  - Every $(s, o, p) \in b - b'$ satisfies the †-property relative to S'
  - Every $(s, o, p) \in b'$ that does not satisfy the †-property relative to S' is not in b
- $\Sigma(R, D, W, z_0)$ is a secure system if z_0 is a secure state and W satisfies the conditions for the simple security condition, the †-property, and the discretionary security property.



Problem

- This system is *clearly* non-secure!
  - Information flows from higher to lower because of the †-property



Discussion

- Role of Basic Security Theorem is to demonstrate that rules preserve security
- Key question: what is security?
  - Bell-LaPadula defines it in terms of 3 properties (simple security condition, *-property, discretionary security property)
  - Theorems are assertions about these properties
  - Rules describe changes to a *particular* system instantiating the model
  - Showing system is secure requires proving rules preserve these 3 properties

Rules and Model

- Nature of rules is irrelevant to model
- Model treats “security” as axiomatic
- Policy defines “security”
  - This instantiates the model
  - Policy reflects the requirements of the systems
- McLean’s definition differs from Bell-LaPadula
  - … and is not suitable for a confidentiality policy
- Analysts cannot prove “security” definition is appropriate through the model

System Z

- System supporting weak tranquility
- On *any* request, system downgrades *all* subjects and objects to lowest level and adds the requested access permission
  - Let initial state satisfy all 3 properties
  - Successive states also satisfy all 3 properties
- Clearly not secure
  - On first request, everyone can read everything

Reformulation of Secure Action

- Given state that satisfies the 3 properties, the action transforms the system into a state that satisfies these properties and eliminates any accesses present in the transformed state that would violate the property in the initial state, then the action is secure
- BST holds with these modified versions of the 3 properties

Reconsider System Z

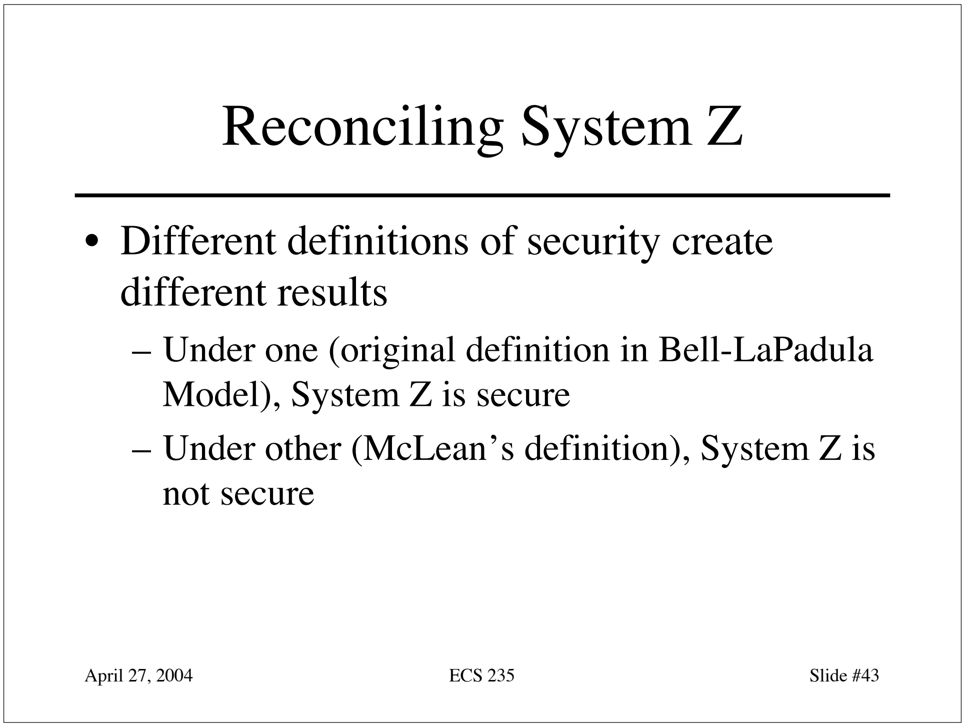
- Initial state has subject s, object o, C = {High, Low}, and K = {All}. Take $f_c(s)$ = (Low, {All}), f_o(o) = (High, {All}), $m[s,o] = \{ \underline{\text{w}} \}$, and $b = \{ (s, o, \underline{\text{w}}) \}$.
- s requests $\underline{\text{r}}$ access to o
- Now $f_o'(o)$ = (Low, {All}), $(s, o, \underline{\text{r}}) \in b'$, and $m[s,o] = \{\underline{\text{r}}, \underline{\text{w}}\}$

Non-Secure System Z

- As $(s, o, \underline{\mathrm{r}}) \in b' - b$ and $f_o(o)$ *dom* $f_c(s)$, access added that was illegal in previous state
  - Under the new version of the Basic Security Theorem, System Z is not secure
  - Under the old version of the Basic Security Theorem, as $f_c'(s) = f_o'(o)$, System Z is secure

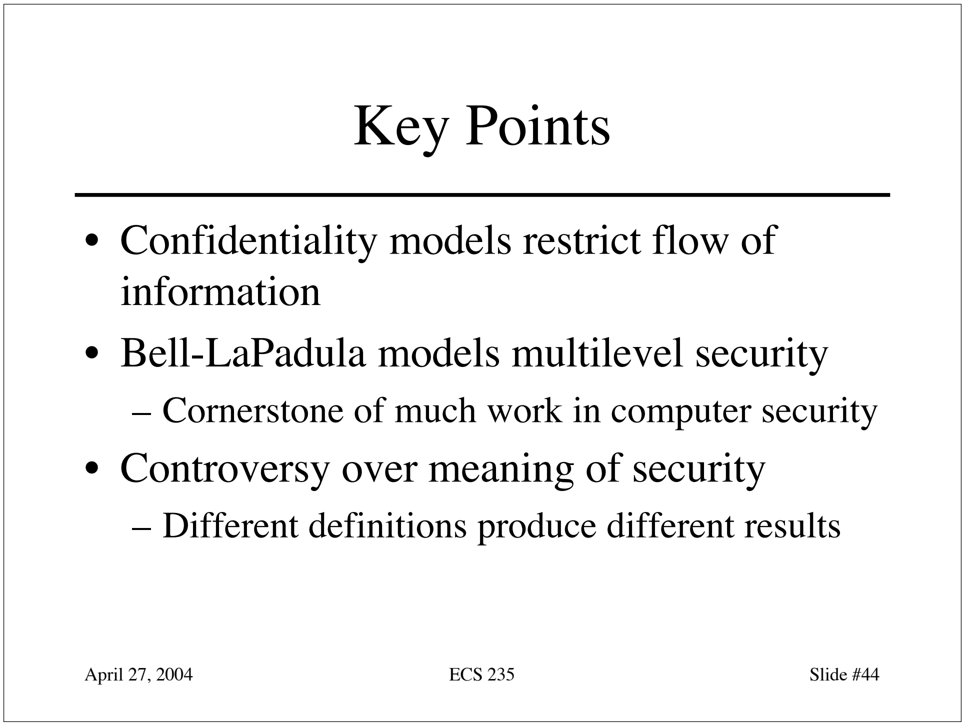
Response: What Is Modeling?

- Two types of models
  1. Abstract physical phenomenon to fundamental properties
  2. Begin with axioms and construct a structure to examine the effects of those axioms
- Bell-LaPadula Model developed as a model in the first sense
  - McLean assumes it was developed as a model in the second sense



Reconciling System Z

- Different definitions of security create different results
  - Under one (original definition in Bell-LaPadula Model), System Z is secure
  - Under other (McLean's definition), System Z is not secure



Key Points

- Confidentiality models restrict flow of information
- Bell-LaPadula models multilevel security
  - Cornerstone of much work in computer security
- Controversy over meaning of security
  - Different definitions produce different results

Overview of Integrity

- Requirements
  - Very different than confidentiality policies
- Biba's models
  - Low-Water-Mark policy
  - Ring policy
  - Strict Integrity policy
- Lipner's model
  - Combines Bell-LaPadula, Biba
- Clark-Wilson model

Requirements of Policies

1. Users will not write their own programs, but will use existing production programs and databases.
2. Programmers will develop and test programs on a nonproduction system; if they need access to actual data, they will be given production data via a special process, but will use it on their development system.
3. A special process must be followed to install a program from the development system onto the production system.
4. The special process in requirement 3 must be controlled and audited.
5. The managers and auditors must have access to both the system state and the system logs that are generated.