Lecture 8
October 7, 2022

Checksums

- Mathematical function to generate a set of *k* bits from a set of *n* bits (where $k \leq n$).
  - *k* is smaller than *n* except in unusual circumstances
- Example: ASCII parity bit
  - ASCII has 7 bits; 8th bit is “parity”
  - Even parity: even number of 1 bits
  - Odd parity: odd number of 1 bits

Example Use

- Bob receives “10111101” as bits.
  - Sender is using even parity; 6 1 bits, so character was received correctly
    - Note: could be garbled, but 2 bits would need to have been changed to preserve parity
  - Sender is using odd parity; even number of 1 bits, so character was not received correctly

Definition of Cryptographic Checksum

Cryptographic checksum h: $A \rightarrow B$:

1. For any $x \in A$, $h(x)$ is easy to compute
2. For any $y \in B$, it is computationally infeasible to find $x \in A$ such that $h(x) = y$
3. It is computationally infeasible to find two inputs x, $x' \in A$ such that $x \neq x'$ and $h(x) = h(x')$
   - Alternate form (stronger): Given any $x \in A$, it is computationally infeasible to find a different $x' \in A$ such that $h(x) = h(x')$.

Collisions

- If $x \neq x'$ and $h(x) = h(x')$, x and x' are a *collision*
  - Pigeonhole principle: if there are n containers for n+1 objects, then at least one container will have at least 2 objects in it.
  - Application: if there are 32 files and 8 possible cryptographic checksum values, at least one value corresponds to at least 4 files

Keys

- Keyed cryptographic checksum: requires cryptographic key
  - AES in chaining mode: encipher message, use last *n* bits. Requires a key to encipher, so it is a keyed cryptographic checksum.
- Keyless cryptographic checksum: requires no cryptographic key
  - SHA-512, SHA-3 are examples; older ones include MD4, MD5, RIPEM, SHA-0, and SHA-1 (methods for constructing collisions are known for these)

HMAC

- Make keyed cryptographic checksums from keyless cryptographic checksums
- *h* keyless cryptographic checksum function that takes data in blocks of *b* bytes and outputs blocks of *l* bytes. *k*′ is cryptographic key of length *b* bytes
  - If short, pad with 0 bytes; if long, hash to length *b*
- *ipad* is 00110110 repeated *b* times
- *opad* is 01011100 repeated *b* times
- HMAC-*h*(*k*, *m*) = *h*(*k*′ ⊕ *opad* || *h*(*k*′ ⊕ *ipad* || *m*))
  - ⊕ exclusive or, || concatenation

Strength of HMAC-*h*

- Depends on the strength of the hash function *h*
- Attacks on HMAC-MD4, HMAC-MD5, HMAC-SHA-0, and HMAC-SHA-1 recover partial or full keys
  - Note all of MD4, MD5, SHA-0, and SHA-1 have been broken

Digital Signature

- Construct that authenticates origin, contents of message in a manner provable to a disinterested third party (a “judge”)
- Sender cannot deny having sent message (service is “nonrepudiation”)
  - Limited to *technical* proofs
    - Inability to deny one’s cryptographic key was used to sign
  - One could claim the cryptographic key was stolen or compromised
    - Legal proofs, *etc.,* probably required; not dealt with here

Common Error

- Symmetric: Alice, Bob share key *k*
  - Alice sends *m* || { *m* } *k* to Bob
  - { *m* } *k* means *m* enciphered with key *k*, || means concatenation

Claim: This is a digital signature

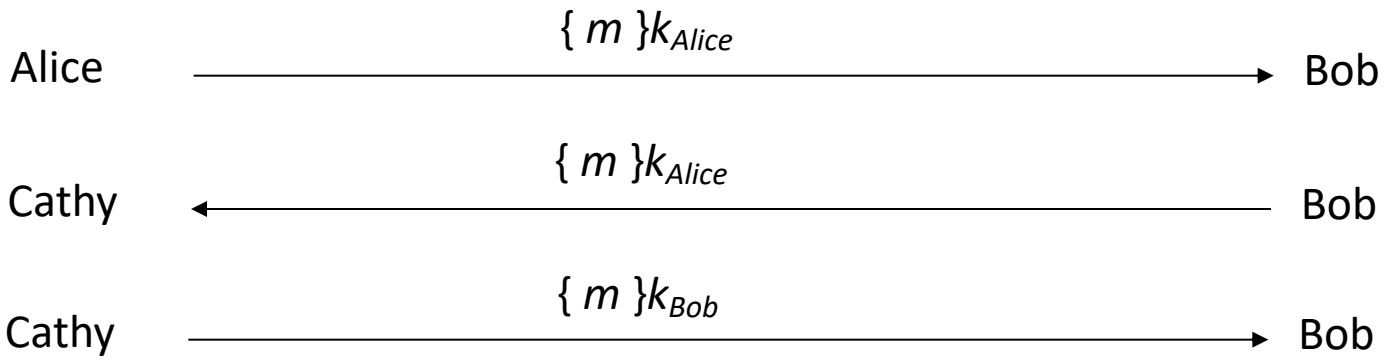
WRONG

This is not a digital signature

- Why? Third party cannot determine whether Alice or Bob generated message

Classical Digital Signatures

- Require trusted third party
  - Alice, Bob each share keys with trusted party Cathy
- To resolve dispute, judge gets $\{ m \} k_{Alice}$, $\{ m \} k_{Bob}$, and has Cathy decipher them; if messages matched, contract was signed



Public Key Digital Signatures

- Basically, Alice enciphers the message, or its cryptographic hash, with her private key
- In case of dispute or question of origin or whether changes have been made, a judge can use Alice's public key to verify the message came from Alice and has not been changed since being signed

RSA Digital Signatures

- Alice's keys are (e_{Alice},n_{Alice}) (public key), d_{Alice} (private key)
  - In what follows, we use e_{Alice} to represent the public key
- Alice sends Bob

$$m \,||\, \{ m \}\, d_{Alice}$$

- In case of dispute, judge computes

$$\{ \{ m \}\, d_{Alice} \}\, e_{Alice}$$

- and if it is m, Alice signed message
  - She's the only one who knows d_{Alice}!

RSA Digital Signatures

- Use private key to encipher message
  - Protocol for use is *critical*
- Key points:
  - Never sign random documents, and when signing, always sign hash and never document
  - Don’t just encipher message and then sign, or vice versa
    - Changing public key and private key can cause problems
    - Messages can be forwarded, so third party cannot tell if original sender sent it to her

Attack #1

- Example: Alice, Bob communicating
  - $n_A = 262631$, $e_A = 154993$, $d_A = 95857$
  - $n_B = 288329$, $e_B = 22579$, $d_B = 138091$
- Alice asks Bob to sign 225536 so she can verify she has the right public key:
  - $c = m^{d_B} \bmod n_B = 225536^{138091} \bmod 288329 = 271316$
- Now she asks Bob to sign the statement AYE (002404):
  - $c = m^{d_B} \bmod n_B = 002404^{138091} \bmod 288329 = 182665$

Attack #1

- Alice computes:
  - new message NAY (130024) by (002404)(225536) mod 288329 = 130024
  - corresponding signature (271316)(182665) mod 288329 = 218646
- Alice now claims Bob signed NAY (130024), and as proof supplies signature 218646
- Judge computes $c^{e_B} \bmod n_B = 218646^{22579} \bmod 288329 = 130024$
  - Signature validated; Bob is toast

Preventing Attack #1

- Do not sign random messages
  - This would prevent Alice from getting the first message
- When signing, always sign the cryptographic hash of a message, not the message itself

Attack #2: Bob's Revenge

- Bob, Alice agree to sign contract LUR (112017)
  - But Bob really wants her to sign contract EWM (042212), but knows she won't
- Alice enciphers, then signs:
  - $(m^{e_B} \bmod n_A)^{d_A} \bmod n_A = (112017^{22579} \bmod 288329)^{95857} \bmod 262631 = 42390$
- Bob now changes his public key
  - Computes r such that $042212^r \bmod 288329 = 112017$; one such $r = 9175$
  - Computes $re_B \bmod \phi(n_B) = (9175)(22579) \bmod 287184 = 102661$
  - Replace public key with (102661,288329), private key with 161245
- Bob claims contract was EWM
- Judge computes:
  - $(42390^{154993} \bmod 262631)^{161245} \bmod 288329 = 042212$, which is EWM
  - Verified; now Alice is toast

Preventing Attack #2

- Obvious thought: instead of encrypting message and then signing it, sign the message and then encrypt it
  - May not work due to surreptitious forwarding attack
  - Idea: Alice sends Cathy an encrypted signed message; Cathy deciphers it, re-enciphers it with Bob's public key, and then sends message and signature to Bob – now Bob thinks the message came from Alice (right) and was intended for him (wrong)
- Several ways to solve this:
  - Put sender and recipient in the message; changing recipient invalidates signature
  - Sign message, encrypt it, then sign the result

El Gamal Digital Signature

- Relies on discrete log problem
  - Choose p prime, g, $d < p$; compute $y = g^d \bmod p$
- Public key: (y, g, p); private key: d
- To sign contract m:
  - Choose k relatively prime to p–1, and not yet used
  - Compute $a = g^k \bmod p$
  - Find b such that $m = (da + kb) \bmod p–1$
  - Signature is (a, b)
- To validate, check that
  - $y^a a^b \bmod p = g^m \bmod p$

Example

- Alice chooses p = 262643, g = 9563, d = 3632, giving y = 274598
- Alice wants to send Bob signed contract PUP (152015)
  - Chooses k = 601 (relatively prime to 262642)
  - This gives $a = g^k \bmod p = 9563^{601} \bmod 29 = 202897$
  - Then solving $152015 = (3632 \times 202897 + 601b) \bmod 262642$ gives b = 225835
  - Alice sends Bob message m = 152015 and signature (a,b) = (202897, 225835)
- Bob verifies signature: $g^m \bmod p = 9563^{152015} \bmod 262643 = 157499$ and $y^a a^b \bmod p = 27459^{202897} 202897^{225835} \bmod 262643 = 157499$
  - They match, so Alice signed

Attack

- Eve learns k, corresponding message m, and signature (a, b)
  - Extended Euclidean Algorithm gives d, the private key
- Example from above: Eve learned Alice signed last message with $k = 5$

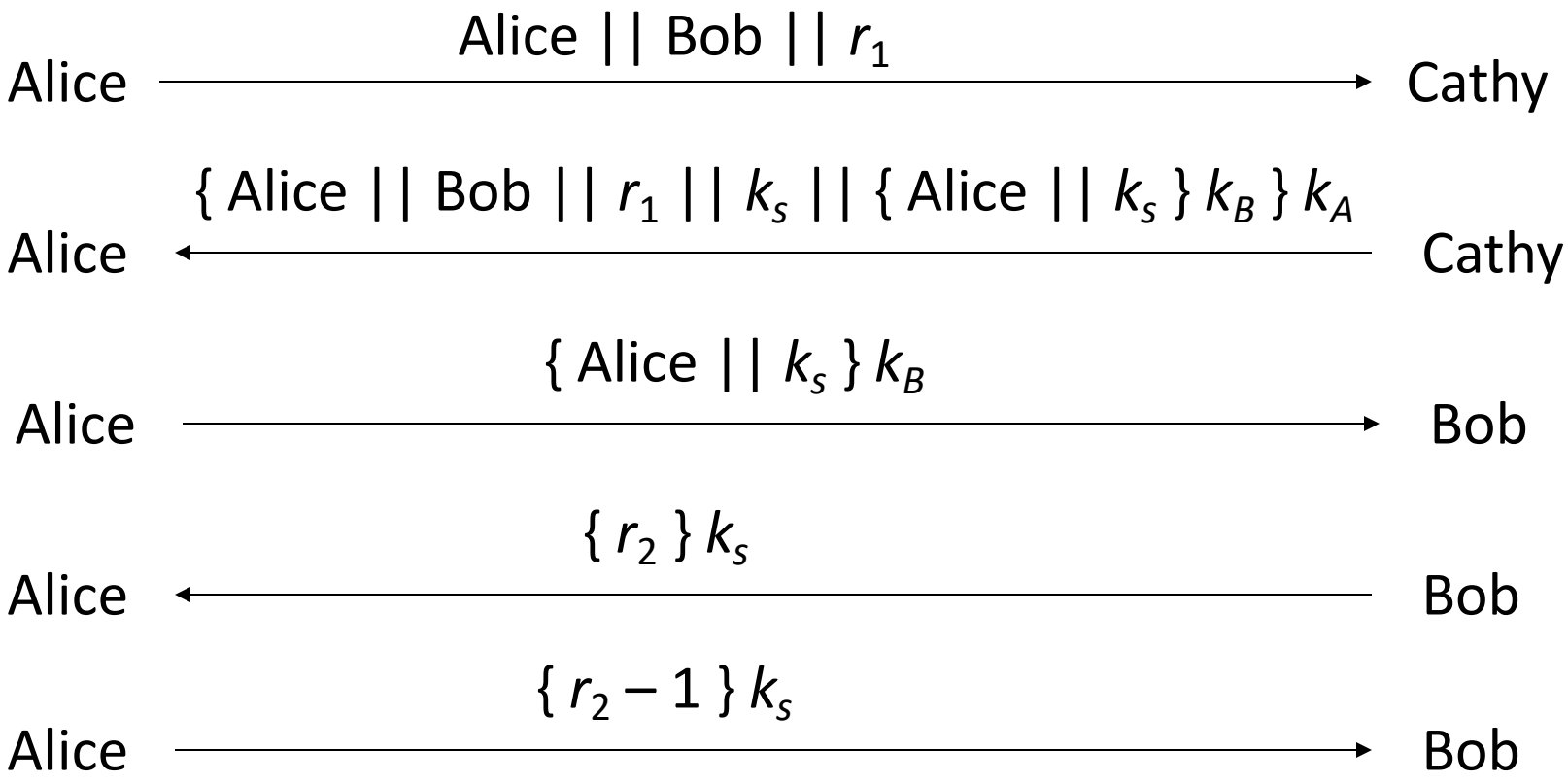
  $$m = (da + kb) \bmod p-1 \Rightarrow 152015 = (202897d + 601 \times 225835) \bmod 262642$$

  giving Alice's private key $d = 3632$

Key Exchange Algorithms

- Goal: Alice, Bob get shared key
  - Key cannot be sent in clear
    - Attacker can listen in
    - Key can be sent enciphered, or derived from exchanged data plus data not known to an eavesdropper
  - Alice, Bob may trust third party
  - All cryptosystems, protocols publicly known
    - Only secret data is the keys, ancillary information known only to Alice and Bob needed to derive keys
    - Anything transmitted is assumed known to attacker

Needham-Schroeder



Argument: Alice talking to Bob

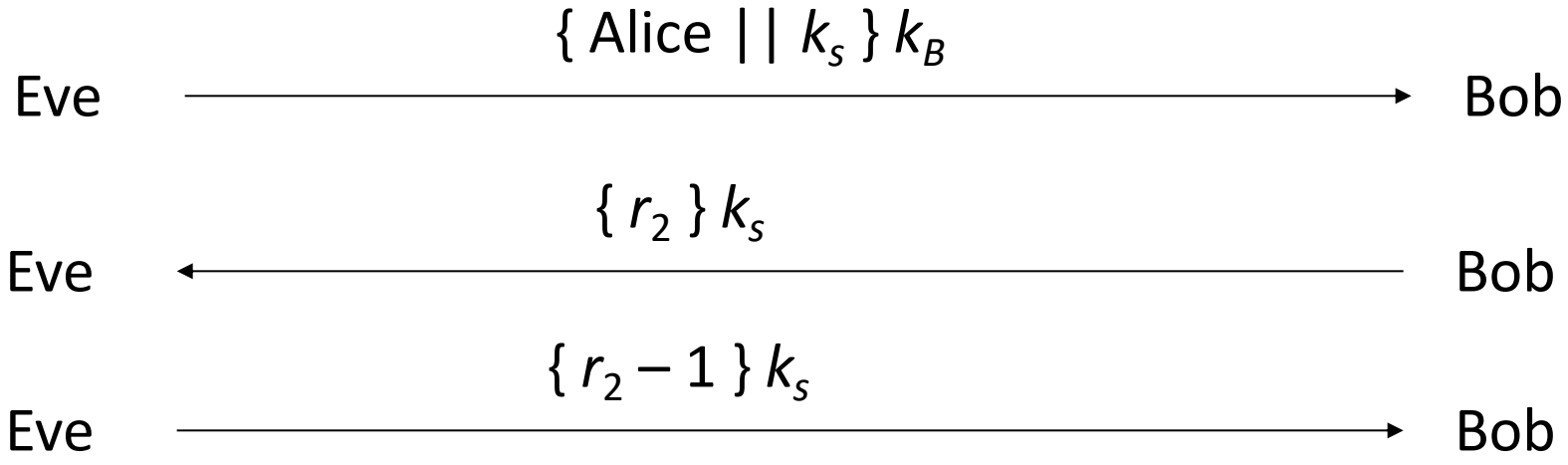
- Second message
  - Enciphered using key only she, Cathy knows
    - So Cathy enciphered it
  - Response to first message
    - As r_1 in it matches r_1 in first message
- Third message
  - Alice knows only Bob can read it
    - As only Bob can derive session key from message
  - Any messages enciphered with that key are from Bob

Argument: Bob talking to Alice

- Third message
  - Enciphered using key only he, Cathy know
    - So Cathy enciphered it
  - Names Alice, session key
    - Cathy provided session key, says Alice is other party
- Fourth message
  - Uses session key to determine if it is replay from Eve
    - If not, Alice will respond correctly in fifth message
    - If so, Eve can't decipher r_2 and so can't respond, or responds incorrectly

Denning-Sacco Modification

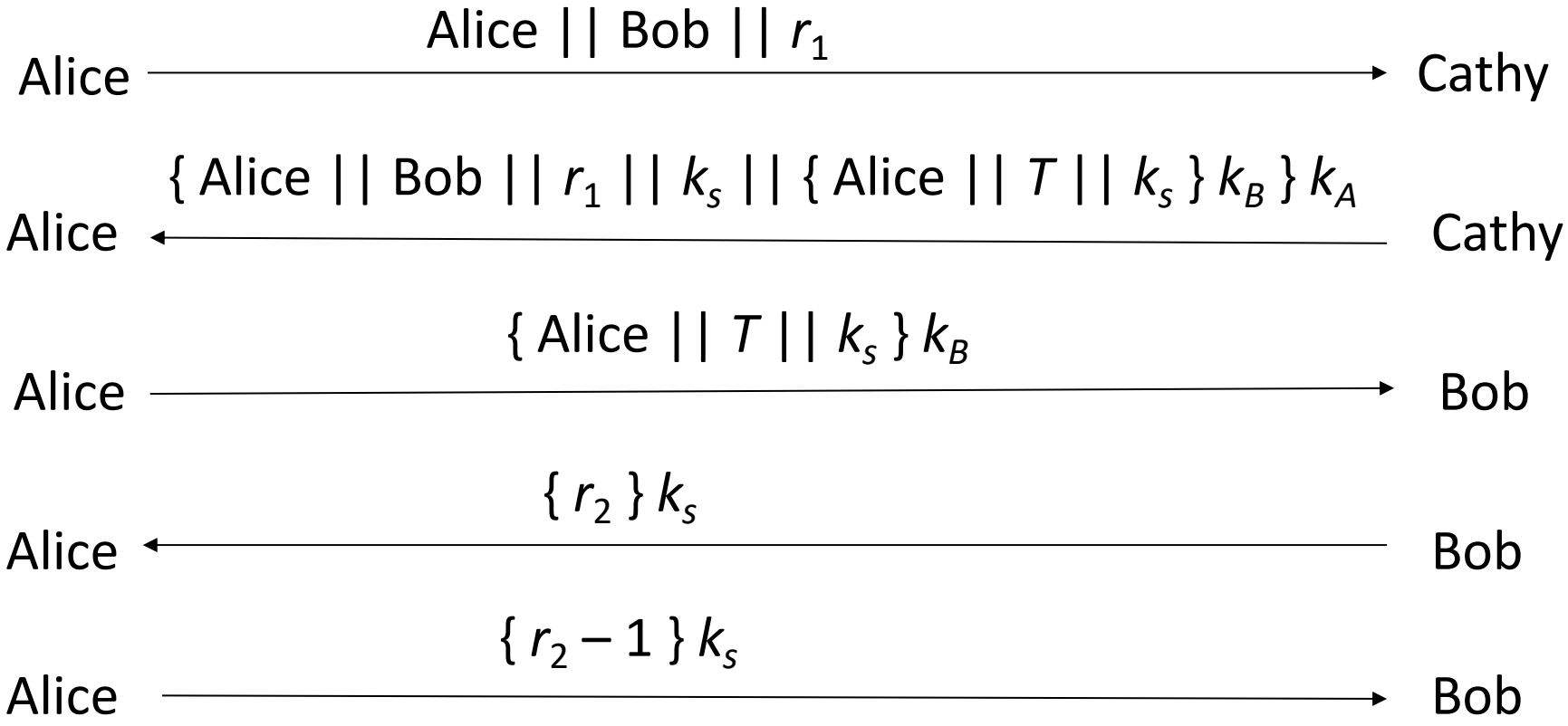
- Assumption: all keys are secret
- Question: suppose Eve can obtain session key. How does that affect protocol?
  - In what follows, Eve knows k_s



Problem and Solution

- In protocol above, Eve impersonates Alice
- Problem: replay in third step
  - First in previous slide
- Solution: use time stamp *T* to detect replay
- Weakness: if clocks not synchronized, may either reject valid messages or accept replays
  - Parties with either slow or fast clocks vulnerable to replay
  - Resetting clock does *not* eliminate vulnerability

Needham-Schroeder with Denning-Sacco Modification



Kerberos

- Authentication system
  - Based on Needham-Schroeder with Denning-Sacco modification
  - Central server plays role of trusted third party (“Cathy”)
- Ticket
  - Issuer vouches for identity of requester of service
- Authenticator
  - Identifies sender

Idea

- User *u* authenticates to Kerberos server
  - Obtains ticket $T_{u,TGS}$ for ticket granting service (TGS)
- User *u* wants to use service *s*:
  - User sends authenticator A_u, ticket $T_{u,TGS}$ to TGS asking for ticket for service
  - TGS sends ticket $T_{u,s}$ to user
  - User sends A_u, $T_{u,s}$ to server as request to use *s*
- Details follow

Ticket

- Credential saying issuer has identified ticket requester
- Example ticket issued to user *u* for service *s*

$$T_{u,s} = s \,||\, \{\, u \,||\, u\text{'s address} \,||\, \text{valid time} \,||\, k_{u,s} \,\}\, k_s$$

where:

- $k_{u,s}$ is session key for user and service
- Valid time is interval for which ticket valid
- *u*'s address may be IP address or something else
  - Note: more fields, but not relevant here

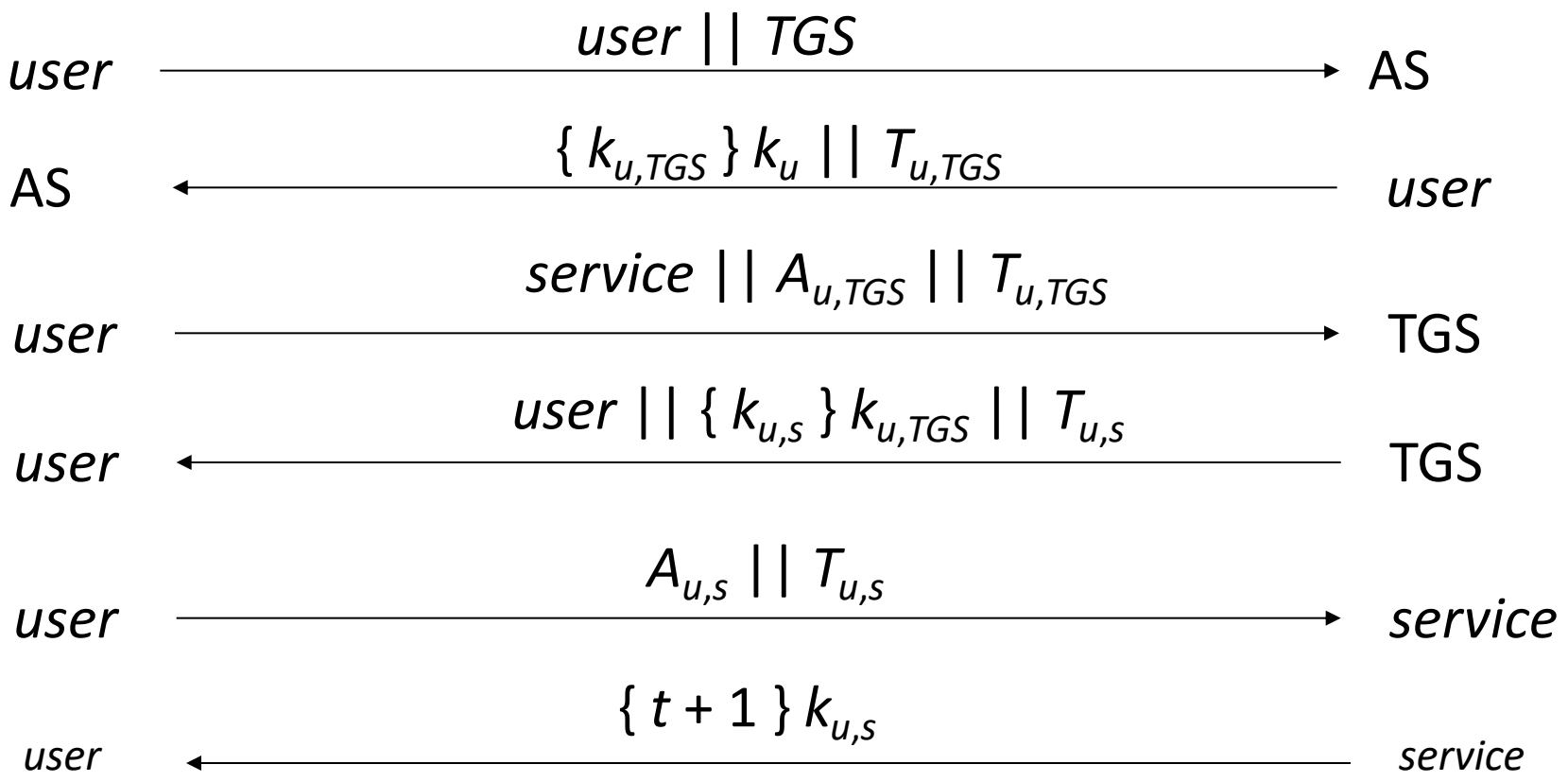
Authenticator

- Credential containing identity of sender of ticket
  - Used to confirm sender is entity to which ticket was issued
- Example: authenticator user *u* generates for service *s*

$$A_{u,s} = \{ u \,||\, \text{generation time} \,||\, k_t \}\, k_{u,s}$$

  where:
  - k_t is alternate session key
  - Generation time is when authenticator generated
    - Note: more fields, not relevant here

Protocol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