

# Lecture 24

## November 16, 2022

# Basics of Information Flow

- Bell-LaPadula Model embodies information flow policy
  - Given compartments  $A, B$ , info can flow from  $A$  to  $B$  iff  $B \text{ dom } A$
- So does Biba Model
  - Given compartments  $A, B$ , info can flow from  $A$  to  $B$  iff  $A \text{ dom } B$
- Variables  $x, y$  assigned compartments  $\underline{x}, \underline{y}$  as well as values
  - Confidentiality (Bell-LaPadula): if  $\underline{x} = A, \underline{y} = B$ , and  $B \text{ dom } A$ , then  $y := x$  allowed but not  $x := y$
  - Integrity (Biba): if  $\underline{x} = A, \underline{y} = B$ , and  $A \text{ dom } B$ , then  $x := y$  allowed but not  $y := x$
- For now, focus on confidentiality (Bell-LaPadula)
  - We'll get to integrity later

# Entropy and Information Flow

- Idea: information flows from  $x$  to  $y$  as a result of a sequence of commands  $c$  if you can deduce information about  $x$  before  $c$  from the value in  $y$  after  $c$
- Formally:
  - $s$  time before execution of  $c$ ,  $t$  time after
  - $H(x_s | y_t) < H(x_s | y_s)$
  - If no  $y$  at time  $s$ , then  $H(x_s | y_t) < H(x_s)$

# Example 1

- Command is  $x := y + z$ ; where:
  - $x$  does not exist initially (that is, has no value)
  - $0 \leq y \leq 7$ , equal probability
  - $z = 1$  with probability  $1/2$ ,  $z = 2$  or  $3$  with probability  $1/4$  each
- $s$  state before command executed;  $t$ , after; so
  - $H(y_s) = H(y_t) = -8(1/8) \lg(1/8) = 3$
- You can show that  $H(y_s | x_t) = (3/32) \lg 3 + 9/8 \approx 1.274 < 3 = H(y_s)$ 
  - Thus, information flows from  $y$  to  $x$

# Example 2

- Command is

**if  $x = 1$  then  $y := 0$  else  $y := 1$ ;**

where  $x, y$  equally likely to be either 0 or 1

- $H(x_s) = 1$  as  $x$  can be either 0 or 1 with equal probability
- $H(x_s | y_t) = 0$  as if  $y_t = 1$  then  $x_s = 0$  and vice versa
  - Thus,  $H(x_s | y_t) = 0 < 1 = H(x_s)$
- So information flowed from  $x$  to  $y$

# Implicit Flow of Information

- Information flows from  $x$  to  $y$  without an *explicit* assignment of the form  $y := f(x)$ 
  - $f(x)$  an arithmetic expression with variable  $x$
- Example from previous slide:  
**if  $x = 1$  then  $y := 0$  else  $y := 1$ ;**
- So must look for implicit flows of information to analyze program

# Notation

- $\underline{x}$  means class of  $x$ 
  - In Bell-LaPadula based system, same as “label of security compartment to which  $x$  belongs”
- $\underline{x} \leq \underline{y}$  means “information can flow from an element in class of  $x$  to an element in class of  $y$ ”
  - Or, “information with a label placing it in class  $\underline{x}$  can flow into class  $\underline{y}$ ”

# Compiler-Based Mechanisms

- Detect unauthorized information flows in a program during compilation
- Analysis not precise, but secure
  - If a flow *could* violate policy (but may not), it is unauthorized
  - No unauthorized path along which information could flow remains undetected
- Set of statements *certified* with respect to information flow policy if flows in set of statements do not violate that policy



# Example

```
if  $x = 1$  then  $y := a$ ;
```

```
else  $y := b$ ;
```

- Information flows from  $x$  and  $a$  to  $y$ , or from  $x$  and  $b$  to  $y$
- Certified only if  $\underline{x} \leq \underline{y}$  and  $\underline{a} \leq \underline{y}$  and  $\underline{b} \leq \underline{y}$ 
  - Note flows for *both* branches must be true unless compiler can determine that one branch will *never* be taken

# Declarations

- Notation:

**$x: \text{int class } \{ A, B \}$**

means  $x$  is an integer variable with security class at least  $\text{lub}\{ A, B \}$ , so  $\text{lub}\{ A, B \} \leq \underline{x}$

- Distinguished classes *Low*, *High*
  - Constants are always *Low*

# Input Parameters

- Parameters through which data passed into procedure
- Class of parameter is class of actual argument

$i_p$ : **type class** {  $i_p$  }

# Output Parameters

- Parameters through which data passed out of procedure
  - If data passed in, called input/output parameter
- As information can flow from input parameters to output parameters, class must include this:

$$o_p: \text{ *type class* } \{ r_1, \dots, r_n \}$$

where  $r_i$  is class of  $i$ th input or input/output argument

# Example

```
proc sum(x: int class { A };  
    var out: int class { A, B } );  
begin  
    out := out + x;  
end;
```

- Require  $\underline{x} \leq \underline{out}$  and  $\underline{out} \leq \underline{out}$

# Array Elements

- Information flowing out:

$$\dots := a[i]$$

Value of  $i$ ,  $a[i]$  both affect result, so class is  $\text{lub}\{\underline{a[i]}, \underline{i}\}$

- Information flowing in:

$$a[i] := \dots$$

- Only value of  $a[i]$  affected, so class is  $\underline{a[i]}$

# Assignment Statements

$x := y + z;$

- Information flows from  $y, z$  to  $x$ , so this requires  $\text{lub}\{\underline{y}, \underline{z}\} \leq \underline{x}$

More generally:

$y := f(x_1, \dots, x_n)$

- the relation  $\text{lub}\{\underline{x}_1, \dots, \underline{x}_n\} \leq \underline{y}$  must hold

# Compound Statements

$x := y + z; a := b * c - x;$

- First statement:  $\text{lub}\{ \underline{y}, \underline{z} \} \leq \underline{x}$
- Second statement:  $\text{lub}\{ \underline{b}, \underline{c}, \underline{x} \} \leq \underline{a}$
- So, both must hold (i.e., be secure)

More generally:

$S_1; \dots; S_n;$

- Each individual  $S_i$  must be secure



# Conditional Statements

`if  $x + y < z$  then  $a := b$  else  $d := b * c - x$ ; end`

- Statement executed reveals information about  $x, y, z$ , so  $\text{lub}\{\underline{x}, \underline{y}, \underline{z}\} \leq \text{glb}\{\underline{a}, \underline{d}\}$

More generally:

`if  $f(x_1, \dots, x_n)$  then  $S_1$  else  $S_2$ ; end`

- $S_1, S_2$  must be secure
- $\text{lub}\{\underline{x}_1, \dots, \underline{x}_n\} \leq \text{glb}\{\underline{y} \mid y \text{ target of assignment in } S_1, S_2\}$

# Iterative Statements

```
while  $i < n$  do begin  $a[i] := b[i]; i := i + 1;$  end
```

- Same ideas as for “if”, but must terminate

More generally:

```
while  $f(x_1, \dots, x_n)$  do  $S;$ 
```

- Loop must terminate;
- $S$  must be secure
- $\text{lub}\{ \underline{x}_1, \dots, \underline{x}_n \} \leq \text{glb}\{ \underline{y} \mid y \text{ target of assignment in } S \}$

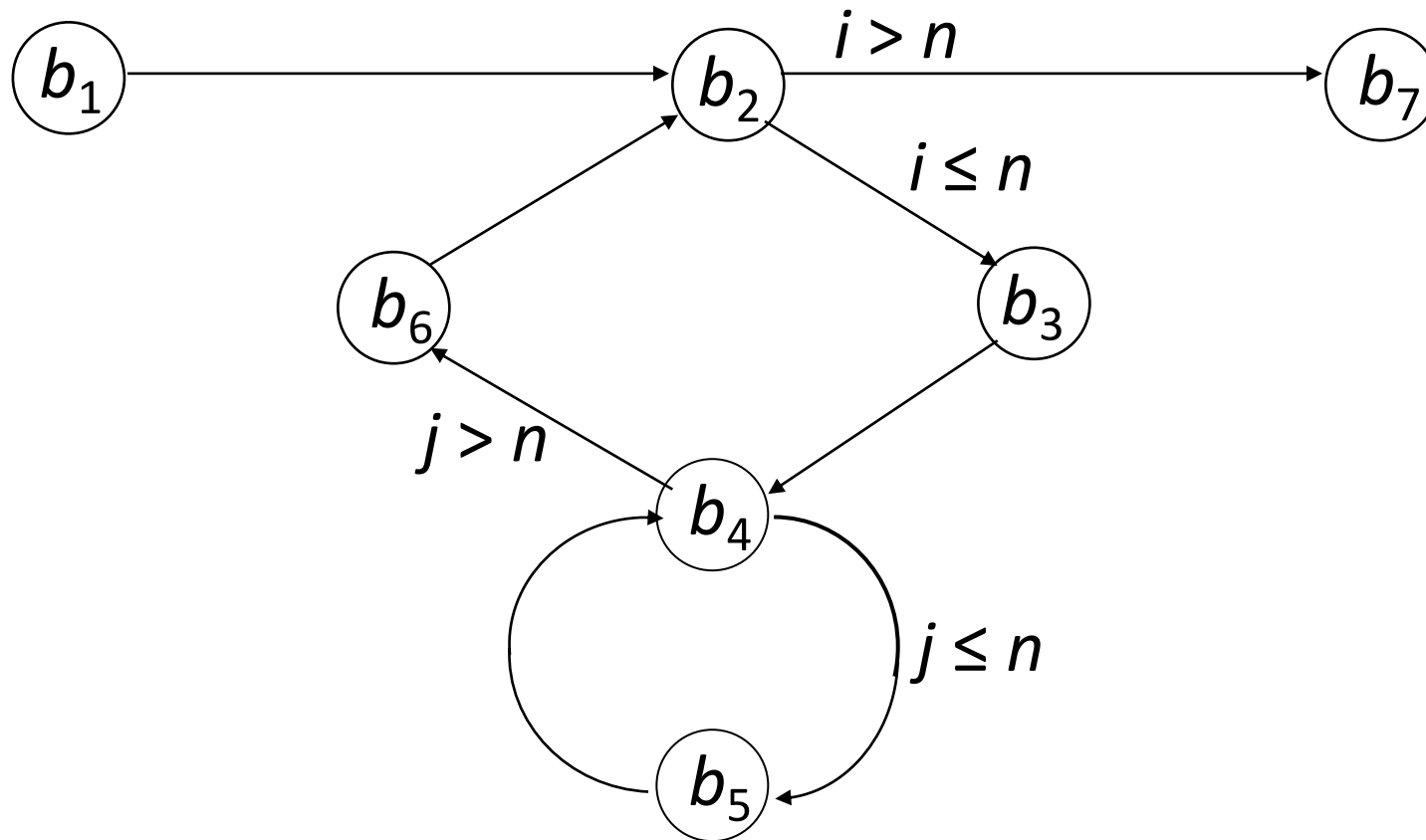
# Goto Statements

- No assignments
  - Hence no explicit flows
- Need to detect implicit flows
- *Basic block* is sequence of statements that have one entry point and one exit point
  - Control in block *always* flows from entry point to exit point

# Example Program

```
proc tm(x: array[1..10][1..10] of integer class {x};  
        var y: array[1..10][1..10] of integer class {y});  
var i, j: integer class {i};  
begin  
b1    i := 1;  
b2 L2: if i > 10 goto L7;  
b3    j := 1;  
b4 L4: if j > 10 then goto L6;  
b5    y[j][i] := x[i][j]; j := j + 1; goto L4;  
b6 L6: i := i + 1; goto L2;  
b7 L7:  
end;
```

# Flow of Control



# Immediate Forward Dominators

- Idea: when two paths out of basic block, implicit flow occurs
  - Because information says *which* path to take
- When paths converge, either:
  - Implicit flow becomes irrelevant; or
  - Implicit flow becomes explicit
- *Immediate forward dominator* of basic block  $b$  (written  $IFD(b)$ ) is first basic block lying on all paths of execution passing through  $b$

# IFD Example

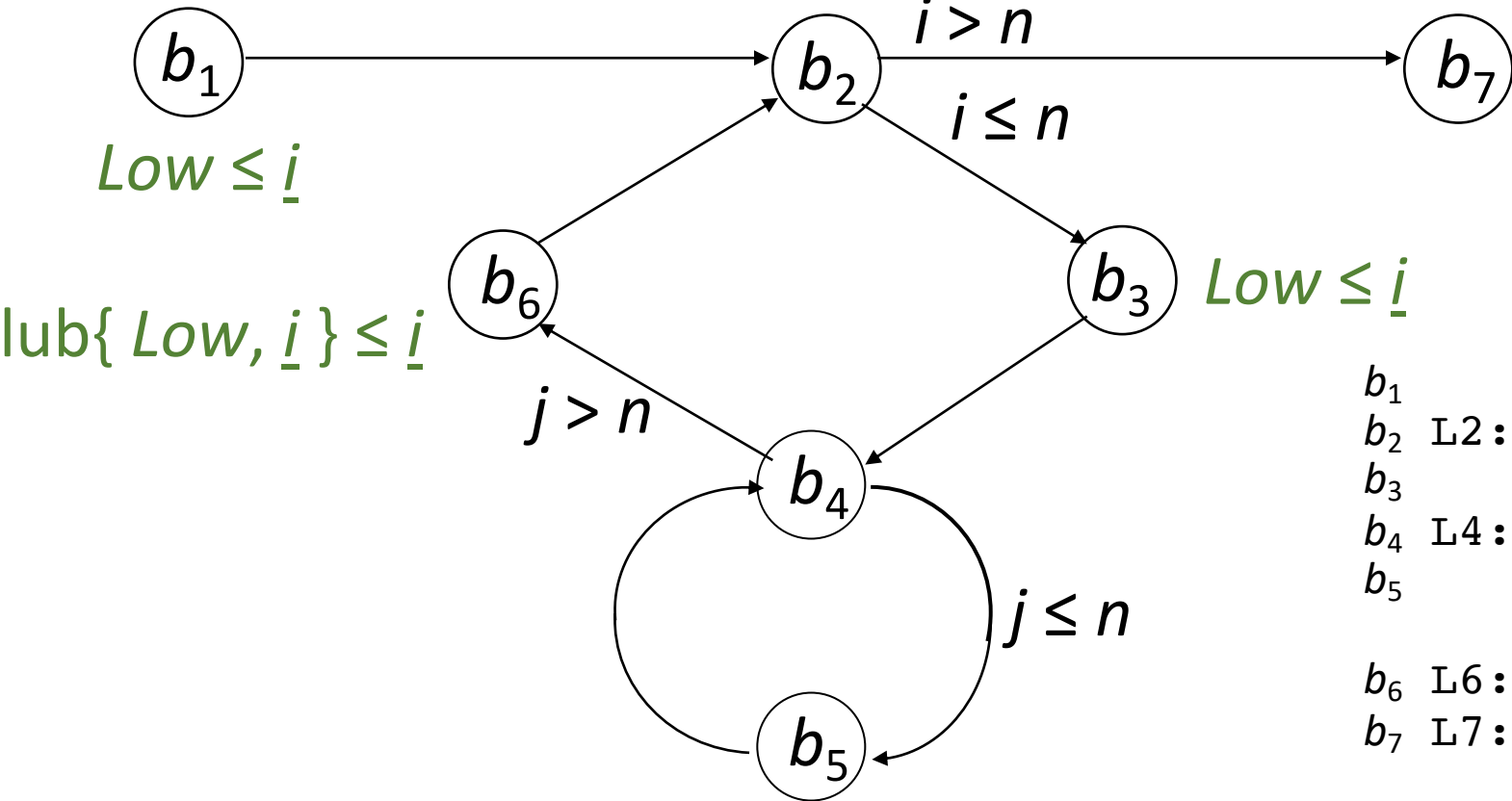
- In previous procedure:
  - $\text{IFD}(b_1) = b_2$       one path
  - $\text{IFD}(b_2) = b_7$        $b_2 \rightarrow b_7$  or  $b_2 \rightarrow b_3 \rightarrow b_6 \rightarrow b_2 \rightarrow b_7$
  - $\text{IFD}(b_3) = b_4$       one path
  - $\text{IFD}(b_4) = b_6$        $b_4 \rightarrow b_6$  or  $b_4 \rightarrow b_5 \rightarrow b_6$
  - $\text{IFD}(b_5) = b_4$       one path
  - $\text{IFD}(b_6) = b_2$       one path

# Requirements

- $B_i$  is set of basic blocks along an execution path from  $b_i$  to  $\text{IFD}(b_i)$ 
  - Analogous to statements in conditional statement
- $x_{i1}, \dots, x_{in}$  variables in expression selecting which execution path containing basic blocks in  $B_i$  used
  - Analogous to conditional expression
- Requirements for secure:
  - All statements in each basic blocks are secure
  - $\text{lub}\{ \underline{x}_{i1}, \dots, \underline{x}_{in} \} \leq \text{glb}\{ \underline{y} \mid y \text{ target of assignment in } B_i \}$



# Example of Requirements



```

b1      i := 1;
b2 L2:  if i > 10 goto L7;
b3      j := 1;
b4 L4:  if j > 10 then goto L6;
b5      y[j][i] := x[i][j];
        j := j + 1; goto L4;
b6 L6:  i := i + 1; goto L2;
b7 L7:
  
```

$\text{lub}\{ \underline{x[i][j]}, i, j \} \leq \underline{y[i][i]} \}; \text{lub}\{ Low, i \} \leq i$

# Example of Requirements

- Within each basic block:

$$b_1: Low \leq \underline{i} \quad b_3: Low \leq \underline{j} \quad b_6: \text{lub}\{Low, \underline{i}\} \leq \underline{i}$$

$$b_5: \text{lub}\{ \underline{x}[\underline{i}][\underline{j}], \underline{i}, \underline{j} \} \leq \underline{y}[\underline{j}][\underline{i}]; \text{lub}\{Low, \underline{i}\} \leq \underline{j}$$

- Combining,  $\text{lub}\{ \underline{x}[\underline{i}][\underline{j}], \underline{i}, \underline{j} \} \leq \underline{y}[\underline{j}][\underline{i}]$
- From declarations, true when  $\text{lub}\{ \underline{x}, \underline{i} \} \leq \underline{y}$
- $B_2 = \{b_3, b_4, b_5, b_6\}$ 
  - Assignments to  $i, j, y[j][i]$ ; conditional is  $i \leq 10$
  - Requires  $\underline{i} \leq \text{glb}\{ \underline{i}, \underline{j}, \underline{y}[\underline{j}][\underline{i}] \}$
  - From declarations, true when  $\underline{i} \leq \underline{y}$

# Example (continued)

- $B_4 = \{ b_5 \}$ 
  - Assignments to  $j, y[j][i]$ ; conditional is  $j \leq 10$
  - Requires  $\underline{j} \leq \text{glb}\{ \underline{j}, \underline{y}[\underline{j}][\underline{i}] \}$
  - From declarations, means  $\underline{j} \leq \underline{y}$
- Result:
  - Combine  $\text{lub}\{ \underline{x}, \underline{i} \} \leq \underline{y}; \underline{i} \leq \underline{y}; \underline{i} \leq \underline{y}$
  - Requirement is  $\text{lub}\{ \underline{x}, \underline{i} \} \leq \underline{y}$

# Procedure Calls

$tm(a, b);$

From previous slides, to be secure,  $\text{lub}\{\underline{x}, \underline{i}\} \leq \underline{y}$  must hold

- In call,  $x$  corresponds to  $a$ ,  $y$  to  $b$
- Means that  $\text{lub}\{\underline{a}, \underline{i}\} \leq \underline{b}$ , or  $\underline{a} \leq \underline{b}$

More generally:

**proc**  $pn(i_1, \dots, i_m: \mathbf{int}; \mathbf{var} \ o_1, \dots, o_n: \mathbf{int}); \mathbf{begin} \ S \ \mathbf{end};$

- $S$  must be secure
- For all  $j$  and  $k$ , if  $\underline{i}_j \leq \underline{o}_k$ , then  $\underline{x}_j \leq \underline{y}_k$
- For all  $j$  and  $k$ , if  $\underline{o}_j \leq \underline{o}_k$ , then  $\underline{y}_j \leq \underline{y}_k$

# Exceptions

```
proc copy(x: integer class { x };  
           var y: integer class Low);  
var sum: integer class { x };  
    z: int class Low;  
begin  
    y := z := sum := 0;  
    while z = 0 do begin  
        sum := sum + x;  
        y := y + 1;  
    end  
end
```

# Exceptions (*cont*)

- When sum overflows, integer overflow trap
  - Procedure exits
  - Value of *sum* is MAXINT/*y*
  - Information flows from *y* to *sum*, but  $\underline{sum} \leq \underline{y}$  never checked
- Need to handle exceptions explicitly
  - Idea: on integer overflow, terminate loop
    - on integer\_overflow\_exception *sum* do *z* := 1;**
  - Now information flows from *sum* to *z*, meaning  $\underline{sum} \leq \underline{z}$
  - This is false ( $\underline{sum} = \{x\}$  dominates  $\underline{z} = \text{Low}$ )

# Infinite Loops

```
proc copy(x: integer 0..1 class { x };  
          var y: integer 0..1 class Low);  
begin  
    y := 0;  
    while x = 0 do  
        (* nothing *);  
    y := 1;  
end
```

- If  $x = 0$  initially, infinite loop
- If  $x = 1$  initially, terminates with  $y$  set to 1
- No explicit flows, but implicit flow from  $x$  to  $y$

# Semaphores

Use these constructs:

```
wait(x):    if x = 0 then block until x > 0; x := x - 1;
```

```
signal(x): x := x + 1;
```

- *x* is semaphore, a shared variable
- Both executed atomically

Consider statement

```
wait(sem); x := x + 1;
```

- Implicit flow from *sem* to *x*
  - Certification must take this into account!



# Flow Requirements

- Semaphores in *signal* irrelevant
  - Don't affect information flow in that process
- Statement  $S$  is a *wait*
  - $\text{shared}(S)$ : set of shared variables read
    - Idea: information flows out of variables in  $\text{shared}(S)$
  - $\text{fglb}(S)$ : glb of assignment targets *following*  $S$
  - So, requirement is  $\text{shared}(S) \leq \text{fglb}(S)$
- $\text{begin } S_1; \dots S_n \text{ end}$ 
  - All  $S_i$  must be secure
  - For all  $i$ ,  $\underline{\text{shared}(S_i)} \leq \text{fglb}(S_i)$

# Example

**begin**

$x := y + z; \quad (* S_1 *)$

$\text{wait}(sem); \quad (* S_2 *)$

$a := b * c - x; \quad (* S_3 *)$

**end**

- Requirements:

- $\text{lub}\{\underline{y}, \underline{z}\} \leq \underline{x}$

- $\text{lub}\{\underline{b}, \underline{c}, \underline{x}\} \leq \underline{a}$

- $\underline{sem} \leq \underline{a}$

- Because  $\text{fglb}(S_2) = \underline{a}$  and  $\text{shared}(S_2) = sem$

# Concurrent Loops

- Similar, but wait in loop affects *all* statements in loop
  - Because if flow of control loops, statements in loop before wait may be executed after wait
- Requirements
  - Loop terminates
  - All statements  $S_1, \dots, S_n$  in loop secure
  - $\text{lub}\{ \underline{\text{shared}}(S_1), \dots, \underline{\text{shared}}(S_n) \} \leq \text{glb}(t_1, \dots, t_m)$ 
    - Where  $t_1, \dots, t_m$  are variables assigned to in loop

# Loop Example

```
while  $i < n$  do begin
```

```
     $a[i] := item;$       ( *  $S_1$  * )
```

```
    wait( $sem$ );        ( *  $S_2$  * )
```

```
     $i := i + 1;$       ( *  $S_3$  * )
```

```
end
```

- Conditions for this to be secure:
  - Loop terminates, so this condition met
  - $S_1$  secure if  $\text{lub}\{ \underline{i}, \underline{item} \} \leq \underline{a[i]}$
  - $S_2$  secure if  $\underline{sem} \leq \underline{i}$  and  $\underline{sem} \leq \underline{a[i]}$
  - $S_3$  trivially secure

# *cobegin/coend*

## **cobegin**

$x := y + z; \quad (* S_1 *)$

$a := b * c - y; \quad (* S_2 *)$

## **coend**

- No information flow among statements
  - For  $S_1$ ,  $\text{lub}\{\underline{y}, \underline{z}\} \leq \underline{x}$
  - For  $S_2$ ,  $\text{lub}\{\underline{b}, \underline{c}, \underline{y}\} \leq \underline{a}$
- Security requirement is both must hold
  - So this is secure if  $\text{lub}\{\underline{y}, \underline{z}\} \leq \underline{x} \wedge \text{lub}\{\underline{b}, \underline{c}, \underline{y}\} \leq \underline{a}$

# Soundness

- Above exposition intuitive
- Can be made rigorous:
  - Express flows as types
  - Equate certification to correct use of types
  - Checking for valid information flows same as checking types conform to semantics imposed by security policy

# Execution-Based Mechanisms

- Detect and stop flows of information that violate policy
  - Done at run time, not compile time
- Obvious approach: check explicit flows
  - Problem: assume for security,  $\underline{x} \leq \underline{y}$   
**if  $x = 1$  then  $y := a$ ;**
  - When  $x \neq 1$ ,  $\underline{x} = \text{High}$ ,  $\underline{y} = \text{Low}$ ,  $\underline{a} = \text{Low}$ , appears okay—but implicit flow violates condition!

# Fenton's Data Mark Machine

- Each variable has an associated class
- Program counter (PC) has one too
- Idea: branches are assignments to PC, so you can treat implicit flows as explicit flows
- Stack-based machine, so everything done in terms of pushing onto and popping from a program stack



# Instruction Description

- *skip*: instruction not executed
- *push*( $x$ ,  $\underline{x}$ ): push variable  $x$  and its security class  $\underline{x}$  onto program stack
- *pop*( $x$ ,  $\underline{x}$ ) : pop top value and security class from program stack, assign them to variable  $x$  and its security class  $\underline{x}$  respectively

# Instructions

- $x := x + 1$  (increment)
  - Same as:  
**if**  $\underline{PC} \leq \underline{x}$  **then**  $x := x + 1$  **else** *skip*
- **if**  $x = 0$  **then goto**  $n$  **else**  $x := x - 1$  (branch and save PC on stack)
  - Same as:  
**if**  $x = 0$  **then begin**  
  **push**( $PC, \underline{PC}$ );  $\underline{PC} := \text{lub}\{\underline{PC}, x\}$ ;  $PC := n$ ;  
**end else if**  $\underline{PC} \leq \underline{x}$  **then**  
   $x := x - 1$   
**else**  
  *skip*;

# More Instructions

- **if'  $x = 0$  then goto  $n$  else  $x := x - 1$**  (branch without saving PC on stack)

- Same as:

```
if  $x = 0$  then
```

```
    if  $\underline{x} \leq \underline{PC}$  then  $PC := n$  else skip
```

```
else
```

```
    if  $\underline{PC} \leq \underline{x}$  then  $x := x - 1$  else skip
```

# More Instructions

- **return** (go to just after last *if*)

- Same as:

**pop**( *PC*, *PC* );

- **halt** (stop)

- Same as:

**if** *program stack empty* **then** *halt*

- Note stack empty to prevent user obtaining information from it after halting

# Goto Statements

- No assignments
  - Hence no explicit flows
- Need to detect implicit flows
- *Basic block* is sequence of statements that have one entry point and one exit point
  - Control in block *always* flows from entry point to exit point