# Lecture #15

- Noninterference
  - Review notation
  - Definition
  - Security policy in these terms
  - Unwinding theorem
  - Example interpretation
  - Dynamic policies
  - Composition

# Security Policy

- Partitions systems into authorized, unauthorized states
- Authorized states have no forbidden interferences
- Hence a *security policy* is a set of noninterference assertions
  - See previous definition

# Alternative Development

- System $X$ is a set of protection domains $D = \{ d_1, \ldots, d_n \}$
- When command $c$ executed, it is executed in protection domain $dom(c)$
- Give alternate versions of definitions shown previously

# Output-Consistency

- $c \in C, dom(c) \in D$
- $\sim^{dom(c)}$ equivalence relation on states of system $X$
- $\sim^{dom(c)}$ *output-consistent* if

  $\sigma_a \sim^{dom(c)} \sigma_b \Rightarrow P(c, \sigma_a) = P(c, \sigma_b)$
- Intuition: states are output-consistent if for subjects in $dom(c)$, projections of outputs for both states after $c$ are the same

# Security Policy

- $D = \{ d_1, \ldots, d_n \}$, $d_i$ a protection domain
- $r$: $D \times D$ a reflexive relation
- Then $r$ defines a security policy
- Intuition: defines how information can flow around a system
  - $d_i r d_j$ means info can flow from $d_i$ to $d_j$
  - $d_i r d_i$ as info can flow within a domain

# Projection Function

- $\pi'$ analogue of $\pi$, earlier
- Commands, subjects absorbed into protection domains
- $d \in D$, $c \in C$, $c_s \in C^*$
- $\pi'_d(\nu) = \nu$
- $\pi'_d(c_s c) = \pi'_d(c_s)c$ if $dom(c) r d$
- $\pi'_d(c_s c) = \pi'_d(c_s)$ otherwise
- Intuition: if executing $c$ interferes with $d$, then $c$ is visible; otherwise, as if $c$ never executed

# Noninterference-Secure

- System has set of protection domains $D$
- System is noninterference-secure with respect to policy $r$ if

$$P^*(c, T^*(c_s, \sigma_0)) = P^*(c, T^*(\pi'_d(c_s), \sigma_0))$$

- Intuition: if executing $c_s$ causes the same transitions for subjects in domain $d$ as does its projection with respect to domain $d$, then no information flows in violation of the policy

# Lemma

- Let $T^*(c_s, \sigma_0) \sim^d T^*(\pi'_d(c_s), \sigma_0)$ for $c \in C$
- If $\sim^{\mathrm{d}}$ output-consistent, then system is noninterference-secure with respect to policy $r$

# Proof

- $d = dom(c)$ for $c \in C$
- By definition of output-consistent,

$$T^*(c_s, \sigma_0) \sim^d T^*(\pi'_d(c_s), \sigma_0)$$

implies

$$P^*(c, T^*(c_s, \sigma_0)) = P^*(c, T^*(\pi'_d(c_s), \sigma_0))$$

- This is definition of noninterference-secure with respect to policy $r$

# Unwinding Theorem

- Links security of sequences of state transition commands to security of individual state transition commands
- Allows you to show a system design is ML secure by showing it matches specs from which certain lemmata derived
  - Says *nothing* about security of system, because of implementation, operation, *etc*. issues

# Locally Respects

- $r$ is a policy
- System $X$ locally respects $r$ if $dom(c)$ being noninterfering with $d \in D$ implies $\sigma_a \sim^d T(c, \sigma_a)$
- Intuition: applying $c$ under policy $r$ to system $X$ has no effect on domain $d$ when $X$ locally respects $r$

# Transition-Consistent

- $r$ policy, d $\in D$
- If $\sigma_a \sim^d \sigma_b$ implies $T(c, \sigma_a) \sim^d T(c, \sigma_b)$, system $X$ transition-consistent under $r$
- Intuition: command $c$ does not affect equivalence of states under policy $r$

# Lemma

- $c_1, c_2 \in C, d \in D$
- For policy $r$, $dom(c_1)rd$ and $dom(c_2)rd$
- Then

$$T^*(c_1c_2,\sigma) = T(c_1,T(c_2,\sigma)) = T(c_2,T(c_1,\sigma))$$

- Intuition: if info can flow from domains of commands into $d$, then order doesn't affect result of applying commands

# Theorem

- $r$ policy, $X$ system that is output consistent, transition consistent, locally respects $r$
- $X$ noninterference-secure with respect to policy $r$
- Significance: basis for analyzing systems claiming to enforce noninterference policy
  - Establish conditions of theorem for particular set of commands, states with respect to some policy, set of protection domains
  - Noninterference security with respect to $r$ follows

# Proof

- Must show $\sigma_a \sim^d \sigma_b$ implies

$$T^*(c_s, \sigma_a) \sim^d T^*(\pi'_d(c_s), \sigma_b)$$

- Induct on length of $c_s$
- Basis: $c_s = \nu$, so $\mathrm{T}^*(c_s, \sigma) = \sigma$; $\pi'_d(\nu) = \nu$; claim holds
- Hypothesis: $c_s = c_1 \ldots c_n$; then claim holds

# Induction Step

- Consider $c_s c_{n+1}$. Assume $\sigma_a \sim^d \sigma_b$ and look at $T^*(\pi'_d(c_s c_{n+1}), \sigma_b)$
- 2 cases:
  - $dom(c_{n+1}) rd$ holds
  - $dom(c_{n+1}) rd$ does not hold

# $dom(c_{n+1})rd$ Holds

$T^*(\pi'_d(c_s c_{n+1}), \sigma_b) = T^*(\pi'_d(c_s)c_{n+1}, \sigma_b)$

$= T(c_{n+1}, T^*(\pi'_d(c_s), \sigma_b))$

– by definition of $T^*$ and $\pi'_d$

- $T(c_{n+1}, \sigma_a) \sim^d T(c_{n+1}, \sigma_b)$
  – as $X$ transition-consistent and $\sigma_a \sim^d \sigma_b$
- $T(c_{n+1}, T^*(c_s, \sigma_a)) \sim^d T(c_{n+1}, T^*(\pi'_d(c_s), \sigma_b))$
  – by transition-consistency and IH

# $dom(c_{n+1})rd$ Holds

$T(c_{n+1}, T^*(c_s, \sigma_a)) \sim^d T(c_{n+1}, T^*(\pi'_d(c_s)c_{n+1}, \sigma_b))$

– by substitution from earlier equality

$T(c_{n+1}, T^*(c_s, \sigma_a)) \sim^d T(c_{n+1}, T^*(\pi'_d(c_s)c_{n+1}, \sigma_b))$

– by definition of $T^*$

- proving hypothesis

# $dom(c_{n+1})rd$ Does Not Hold

$T^*(\pi'_d(c_s c_{n+1}), \sigma_b) = T^*(\pi'_d(c_s), \sigma_b)$

– by definition of $\pi'_d$

$T^*(c_s, \sigma_b) = T^*(\pi'_d(c_s c_{n+1}), \sigma_b)$

– by above and IH

$T(c_{n+1}, T^*(c_s, \sigma_a)) \sim^d T^*(c_s, \sigma_a)$

– as $X$ locally respects $r$, so $\sigma \sim^d T(c_{n+1}, \sigma)$ for any $\sigma$

$T(c_{n+1}, T^*(c_s, \sigma_a)) \sim^d T(c_{n+1}, T^*(\pi'_d(c_s) c_{n+1}, \sigma_b))$

– substituting back

- proving hypothesis

# Finishing Proof

- Take $\sigma_a = \sigma_b = \sigma_0$, so from claim proved by induction,

$$T^*(c_s, \sigma_0) \sim^d T^*(\pi'_d(c_s), \sigma_0)$$

- By previous lemma, as $X$ (and so $\sim^d$) output consistent, then $X$ is noninterference-secure with respect to policy $r$

# Access Control Matrix

- Example of interpretation
- Given: access control information
- Question: are given conditions enough to provide noninterference security?
- Assume: system in a particular state
  - Encapsulates values in ACM

# ACM Model

- Objects $L = \{ l_1, \ldots, l_m \}$
  - Locations in memory
- Values $V = \{ v_1, \ldots, v_n \}$
  - Values that L can assume
- Set of states $\Sigma = \{ \sigma_1, \ldots, \sigma_k \}$
- Set of protection domains $D = \{ d_1, \ldots, d_j \}$

# Functions

- *value*: $L \times \Sigma \rightarrow V$
  - returns value $v$ stored in location $l$ when system in state $\sigma$
- *read*: $D \rightarrow 2^V$
  - returns set of objects observable from domain $d$
- *write*: $D \rightarrow 2^V$
  - returns set of objects observable from domain $d$

# Interpretation of ACM

- Functions represent ACM
  - Subject $s$ in domain $d$, object $o$
  - $r \in A[s, o]$ if $o \in read(d)$
  - $w \in A[s, o]$ if $o \in write(d)$
- Equivalence relation:

$$[\sigma_a \sim^{dom(c)} \sigma_b] \Leftrightarrow [\ \forall l_i \in read(d)\ [\ value(l_i, \sigma_a) = value(l_i, \sigma_b)\ ]\ ]$$

  - You can read the *exactly* the same locations in both states

# Enforcing Policy *r*

- 5 requirements
  - 3 general ones describing dependence of commands on rights over input and output
    - Hold for all ACMs and policies
  - 2 that are specific to some security policies
    - Hold for *most* policies

# Enforcing Policy *r*: First

- Output of command *c* executed in domain *dom*(*c*) depends only on values for which subjects in *dom*(*c*) have read access

$$\sigma_a \sim^{dom(c)} \sigma_b \Rightarrow P(c, \sigma_a) = P(c, \sigma_b)$$

# Enforcing Policy *r*: Second

- If $c$ changes $l_i$, then $c$ can only use values of objects in *read*(*dom*($c$)) to determine new value

$$[\ \sigma_a \sim^{dom(c)} \sigma_b \textit{ and}$$
$$(value(l_i, T(c, \sigma_a)) \neq value(l_i, \sigma_a) \textit{ or}$$
$$value(l_i, T(c, \sigma_b)) \neq value(l_i, \sigma_b))\ ] \Rightarrow$$
$$value(l_i, T(c, \sigma_a)) = value(l_i, T(c, \sigma_b))$$

# Enforcing Policy *r*: Third

- If $c$ changes $l_i$, then $dom(c)$ provides subject executing $c$ with write access to $l_i$

$$value(l_i, T(c, \sigma_a)) \neq value(l_i, \sigma_a) \Rightarrow$$
$$l_i \in write(dom(c))$$

# Enforcing Policies $r$: Fourth

- If domain $u$ can interfere with domain $v$, then every object that can be read in $u$ can also be read in $v$
- So if object $o$ cannot be read in $u$, but can be read in $v$; and object $o'$ in $u$ can be read in $v$, then info flows from $o$ to $o'$, then to $v$

Let $u, v \in D$; then $urv \Rightarrow read(u) \subseteq read(v)$

# Enforcing Policies r: Fifth

- Subject $s$ can write object $o$ in $v$, subject $s'$ can read $o$ in $u$, then domain $v$ can interfere with domain $u$

$$l_i \in read(u) \; and \; l_i \in write(v) \Rightarrow vru$$

# Theorem

- Let *X* be a system satisfying the five conditions. The *X* is noninterference-secure with respect to r
- Proof: must show *X* output-consistent, locally respects *r*, transition-consistent
  – Then by unwinding theorem, theorem holds

# Output-Consistent

- Take equivalence relation to be $\sim^d$, first condition *is* definition of output-consistent

# Locally Respects $r$

- Proof by contradiction: assume $(dom(c),d) \notin r$ but $\sigma_a \sim^d T(c, \sigma_a)$ does not hold
- Some object has value changed by $c$:

  $$\exists\, l_i \in read(d)\ [\ value(l_i, \sigma_a) \neq \text{value}(l_i, T(c, \sigma_a))\ ]$$

- Condition 3: $l_i \in write(d)$
- Condition 5: $dom(c) r d$, contradiction
- So $\sigma_a \sim^d T(c, \sigma_a)$ holds, meaning $X$ locally respects $r$

# Transition Consistency

- Assume $\sigma_a \sim^d \sigma_b$
- Must show $\text{value}(l_i, T(c, \sigma_a)) = value(l_i, T(c, \sigma_b))$ for $l_i \in read(d)$
- 3 cases dealing with change that $c$ makes in $l_i$ in states $\sigma_a, \sigma_b$

# Case 1

- $value(l_i, T(c, \sigma_a)) \neq value(l_i, \sigma_a)$
- Condition 3: $l_i \in write(dom(c))$
- As $l_i \in read(d)$, condition 5 says $dom(c) r d$
- Condition 4 says $read(dom(c)) \subseteq read(d)$
- As $\sigma_a \sim^d \sigma_b$, $\sigma_a \sim^{dom(c)} \sigma_b$
- Condition 2:

$$value(l_i, T(c, \sigma_a)) = value(l_i, T(c, \sigma_b))$$

- So $T(c, \sigma_a) \sim^{dom(c)} T(c, \sigma_b)$, as desired

# Case 2

- $value(l_i, T(c, \sigma_b)) \neq value(l_i, \sigma_b)$
- Condition 3: $l_i \in write(dom(c))$
- As $l_i \in read(d)$, condition 5 says $dom(c) r d$
- Condition 4 says $read(dom(c)) \subseteq read(d)$
- As $\sigma_a \sim^d \sigma_b$, $\sigma_a \sim^{dom(c)} \sigma_b$
- Condition 2:

$$value(l_i, T(c, \sigma_a)) = value(l_i, T(c, \sigma_b))$$

- So $T(c, \sigma_a) \sim^{dom(c)} T(c, \sigma_b)$, as desired

# Case 3

- Neither of the previous two
  - $value(l_i, T(c, \sigma_a)) = value(l_i, \sigma_a)$
  - $value(l_i, T(\mathrm{c}, \sigma_b)) = value(l_i, \sigma_b)$
- Interpretation of $\sigma_a \sim^d \sigma_b$ is:

  for $l_i \in read(d)$, $value(l_i, \sigma_a) = value(l_i, \sigma_b)$
- So $T(c, \sigma_a) \sim^d T(c, \sigma_b)$, as desired
- In all 3 cases, $X$ transition-consistent

# Policies Changing Over Time

- Problem: previous analysis assumes static system
  - In real life, ACM changes as system commands issued
- Example: $w \in C^*$ leads to current state
  - *cando*($w$, $s$, $z$) holds if $s$ can execute $z$ in current state
  - Condition noninterference on *cando*
  - If $\neg$*cando*($w$, Lara, "write $f$"), Lara can't interfere with any other user by writing file $f$

# Generalize Noninterference

- $G \subseteq S$ group of subjects, $A \subseteq Z$ set of commands, $p$ predicate over elements of $C^*$
- $c_s = (c_1, \ldots, c_n) \in C^*$
- $\pi''(\nu) = \nu$
- $\pi''((c_1, \ldots, c_n)) = (c_1', \ldots, c_n')$
  - $c_i' = \nu$ if $p(c_1', \ldots, c_{i-1}')$ and $c_i = (s, z)$ with $s \in G$ and $z \in A$
  - $c_i' = c_i$ otherwise

# Intuition

- $\pi''(c_s) = c_s$
- But if $p$ holds, and element of $c_s$ involves both command in $A$ and subject in $G$, replace corresponding element of $c_s$ with empty command $\nu$
  - Just like deleting entries from $c_s$ as $\pi_{A,G}$ does earlier

# Noninterference

- $G, G' \subseteq S$ groups of subjects, $A \subseteq Z$ set of commands, $p$ predicate over $C^*$
- Users in $G$ executing commands in $A$ are noninterfering with users in $G'$ under condition $p$ iff, for all $c_s \in C^*$, all $s \in G'$, $proj(s, c_s, \sigma_i) = proj(s, \text{p''}(c_s), \sigma_i)$
  – Written $A,G$ :| $G'$ **if** $p$

# Example

- From earlier one, simple security policy based on noninterference:

$\forall (s \in S)\ \forall (z \in Z)$

$[\ \{z\}, \{s\} :\mid S \textbf{ if } \neg cando(w, s, z)\ ]$

- If subject can't execute command (the $\neg$*cando* part), subject can't use that command to interfere with another subject

# Another Example

- Consider system in which rights can be passed
  - $pass(s, z)$ gives $s$ right to execute $z$
  - $w_n = v_1, \ldots, v_n$ sequence of $v_i \in C^*$
  - $prev(w_n) = w_{n-1}$; last($wn$) = $v_n$

# Policy

- No subject $s$ can use $z$ to interfere if, in previous state, $s$ did not have right to $z$, and no subject gave it to $s$

$\{ z \}, \{ s \}$ :| $S$ **if**

$$[\ \neg cando(prev(w), s, z) \wedge$$
$$[\ cando(prev(w), s', pass(s, z)) \Rightarrow$$
$$\neg last(w) = (s', pass(s, z))\ ]\ ]$$

# Effect

- Suppose $s_1 \in S$ can execute $pass(s_2, z)$
- For all $w \in C^*$, $cando(w, s_1, pass(s_2, z))$ true
- Initially, $cando(\nu, s_2, z)$ false
- Let $z' \in Z$ be such that $(s_3, z')$ noninterfering with $(s_2, z)$
  - So for each $w_n$ with $v_n = (s_3, z')$,

$$cando(w_n, s_2, z) = cando(w_{n-1}, s_2, z)$$

# Effect

- Then policy says for all $s \in S$

$$proj(s, ((s_2, z), (s_1, pass(s_2, z)),$$
$$(s_3, z'), (s_2, z)), \sigma_i) =$$
$$proj(s, ((s_1, pass(s_2, z)), (s_3, z'), (s_2, \mathrm{z})), \sigma_i)$$

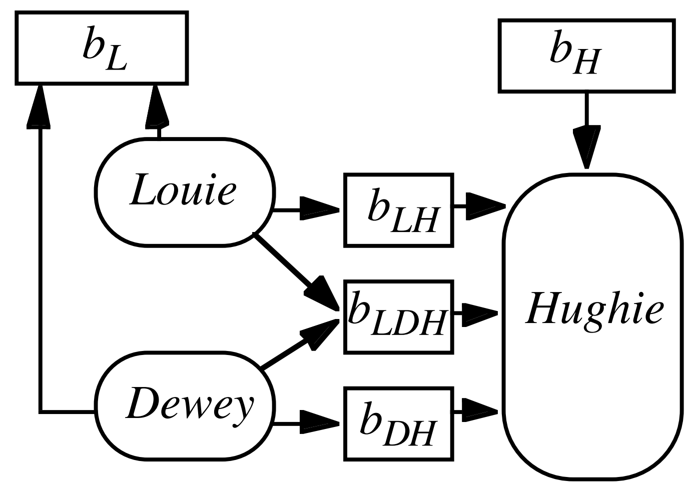
- So $s_2$'s first execution of $z$ does not affect any subject's observation of system

# Policy Composition I

- Assumed: Output function of input
  - Means deterministic (else not function)
  - Means uninterruptability (differences in timings can cause differences in states, hence in outputs)
- This result for deterministic, noninterference-secure systems

# Compose Systems

- Louie, Dewey LOW
- Hughie HIGH
- $b_L$ output buffer
  - Anyone can read it
- $b_H$ input buffer
  - From HIGH source
- Hughie reads from:
  - $b_{LH}$ (Louie writes)
  - $b_{LDH}$ (Louie, Dewey write)
  - $b_{DH}$ (Dewey writes)



# Systems Secure

- All noninterference-secure
  - Hughie has no output
    - So inputs don't interfere with it
  - Louie, Dewey have no input
    - So (nonexistent) inputs don't interfere with outputs