

# Lecture #16

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- Composition
- Nondeducibility
- Generalized Noninterference
- Restrictiveness

# Policy Composition I

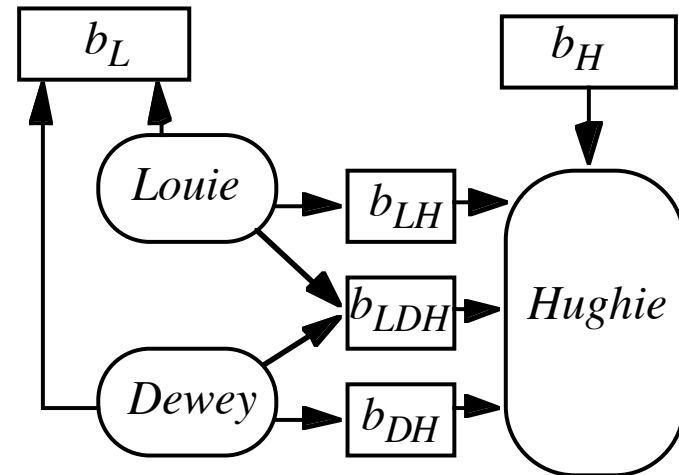
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- Assumed: Output function of input
  - Means deterministic (else not function)
  - Means uninterruptability (differences in timings can cause differences in states, hence in outputs)
- This result for deterministic, noninterference-secure systems

# Compose Systems

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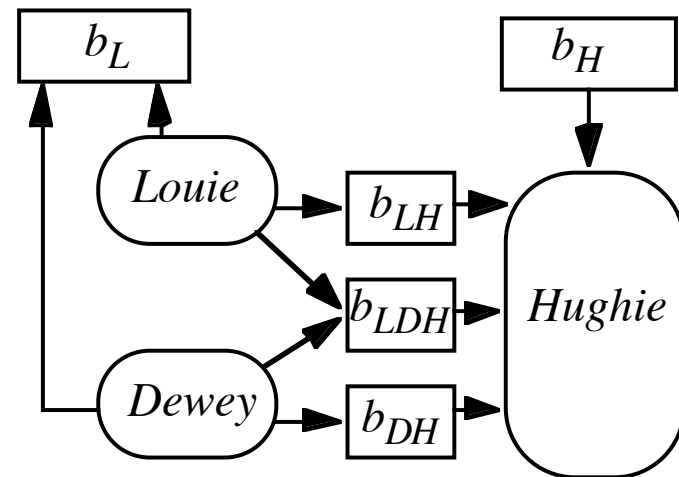
- Louie, Dewey LOW
- Hughie HIGH
- $b_L$  output buffer
  - Anyone can read it
- $b_H$  input buffer
  - From HIGH source
- Hughie reads from:
  - $b_{LH}$  (Louie writes)
  - $b_{LDH}$  (Louie, Dewey write)
  - $b_{DH}$  (Dewey writes)



# Systems Secure

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- All noninterference-secure
  - Hughie has no output
    - So inputs don't interfere with it
  - Louie, Dewey have no input
    - So (nonexistent) inputs don't interfere with outputs



# Security of Composition

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- Buffers finite, sends/receives blocking: composition *not* secure!
  - Example: assume  $b_{DH}$ ,  $b_{LH}$  have capacity 1
- Algorithm:
  1. Louie (Dewey) sends message to  $b_{LH}$  ( $b_{DH}$ )
    - Fills buffer
  2. Louie (Dewey) sends second message to  $b_{LH}$  ( $b_{DH}$ )
  3. Louie (Dewey) sends a 0 (1) to  $b_L$
  4. Louie (Dewey) sends message to  $b_{LDH}$ 
    - Signals Hughie that Louie (Dewey) completed a cycle

# Hughie

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- Reads bit from  $b_H$ 
  - If 0, receive message from  $b_{LH}$
  - If 1, receive message from  $b_{DH}$
- Receive on  $b_{LDH}$ 
  - To wait for buffer to be filled

# Example

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- Hughie reads 0 from  $b_H$ 
  - Reads message from  $b_{LH}$
- Now Louie's second message goes into  $b_{LH}$ 
  - Louie completes setp 2 and writes 0 into  $b_L$
- Dewey blocked at step 1
  - Dewey cannot write to  $b_L$
- Symmetric argument shows that Hughie reading 1 produces a 1 in  $b_L$
- So, input from  $b_H$  copied to output  $b_L$

# Nondeducibility

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- Noninterference: do state transitions caused by high level commands interfere with sequences of state transitions caused by low level commands?
- Really case about inputs and outputs:
  - Can low level subject deduce *anything* about high level outputs from a set of low level outputs?



# Example: 2-Bit System

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- *High* operations change only *High* bit
  - Similar for *Low*
- $\sigma_0 = (0, 0)$
- Commands (Heidi, *xor1*), (Lara, *xor0*), (Lara, *xor1*), (Lara, *xor0*), (Heidi, *xor1*), (Lara, *xor0*)
  - Both bits output after each command
- Output is: 00101011110101

# Security

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- Not noninterference-secure w.r.t. Lara
  - Lara sees output as 0001111
  - Delete *High* and she sees 00111
- But Lara still cannot deduce the commands deleted
  - Don't affect values; only lengths
- So it is deducibly secure
  - Lara can't deduce the commands Heidi gave

# Event System

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- 4-tuple  $(E, I, O, T)$ 
  - $E$  set of events
  - $I \subseteq E$  set of input events
  - $O \subseteq E$  set of output events
  - $T$  set of all finite sequences of events legal within system
- $E$  partitioned into  $H, L$ 
  - $H$  set of *High* events
  - $L$  set of *Low* events

# More Events ...

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- $H \cap I$  set of *High* inputs
- $H \cap O$  set of *High* outputs
- $L \cap I$  set of *Low* inputs
- $L \cap O$  set of *Low* outputs
- $T_{Low}$  set of all possible sequences of *Low* events that are legal within system
- $\pi_L: T \rightarrow T_{Low}$  projection function deleting all *High* inputs from trace
  - *Low* observer should not be able to deduce anything about *High* inputs from trace  $t_{Low} \in T_{low}$

# Deducibly Secure

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- System deducibly secure if, for every trace  $t_{Low} \in T_{Low}$ , the corresponding set of high level traces contains every possible trace  $t \in T$  for which  $\pi_L(t) = t_{Low}$ 
  - Given any  $t_{Low}$ , the trace  $t \in T$  producing that  $t_{Low}$  is equally likely to be *any* trace with  $\pi_L(t) = t_{Low}$

# Example

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- Back to our 2-bit machine
  - Let xor0, xor1 apply to both bits
  - Both bits output after each command
- Initial state: (0, 1)
- Inputs:  $1_H 0_L 1_L 0_H 1_L 0_L$
- Outputs: 10 10 01 01 10 10
- Lara (at *Low*) sees: 001100
  - Does not know initial state, so does not know first input; but can deduce fourth input is 0
- Not deducibly secure

# Example

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- Now  $xor0, xor1$  apply only to state bit with same level as user
- Inputs:  $1_H 0_L 1_L 0_H 1_L 0_L$
- Outputs: 101111011
- Lara sees: 01101
- She cannot deduce *anything* about input
  - Could be  $0_H 0_L 1_L 0_H 1_L 0_L$  or  $0_L 1_H 1_L 0_H 1_L 0_L$  for example
- Deducibly secure

# Security of Composition

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- In general: deducibly secure systems not composable
- *Strong noninterference*: deducible security + requirement that no *High* output occurs unless caused by a *High* input
  - Systems meeting this property *are* composable



# Example

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- 2-bit machine done earlier does not exhibit strong noninterference
  - Because it puts out *High* bit even when there is no *High* input
- Modify machine to output only state bit at level of latest input
  - *Now* it exhibits strong noninterference

# Problem

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- Too restrictive; it bans some systems that are *obviously* secure
- Example: System *upgrade* reads *Low* inputs, outputs those bits at *High*
  - Clearly deducibly secure: low level user sees no outputs
  - Clearly does not exhibit strong noninterference, as no high level inputs!

# Remove Determinism

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- Previous assumption
  - Input, output synchronous
  - Output depends only on commands triggered by input
    - Sometimes absorbed into commands ...
  - Input processed one datum at a time
- Not realistic
  - In real systems, lots of asynchronous events

# Generalized Noninterference

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- Nondeterministic systems meeting noninterference property meet *generalized noninterference-secure property*
  - More robust than nondeducible security because minor changes in assumptions affect whether system is nondeducibly secure

# Example

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- System with *High* Holly, *Low* Lucy, text file at *High*
  - File fixed size, symbol b marks empty space
  - Holly can edit file, Lucy can run this program:

```
while true do begin  
  n := read_integer_from_user;  
  if n > file_length or char_in_file[n] = b then  
    print random_character;  
  else  
    print char_in_file[n];  
end;
```

# Security of System

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- Not noninterference-secure
  - High level inputs—Holly’s changes—affect low level outputs
- *May* be deducibly secure
  - Can Lucy deduce contents of file from program?
  - If output meaningful (“This is right”) or close (“This is right”), yes
  - Otherwise, no
- So deducibly secure depends on which inferences are allowed

# Composition of Systems

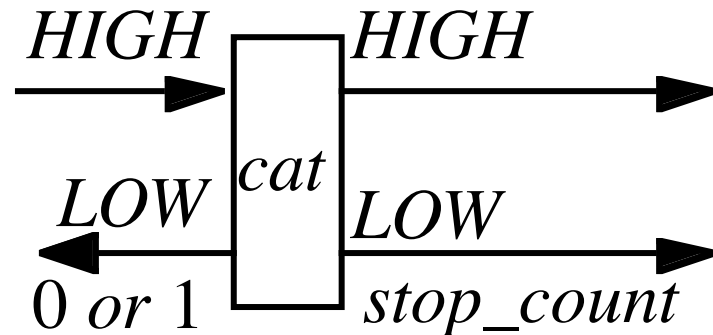
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- Does composing systems meeting generalized noninterference-secure property give you a system that also meets this property?
- Define two systems (*cat*, *dog*)
- Compose them

# First System: *cat*

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- Inputs, outputs can go left or right
- After some number of inputs, *cat* sends two outputs
  - First *stop\_count*
  - Second parity of *High* inputs, outputs





# Noninterference-Secure?

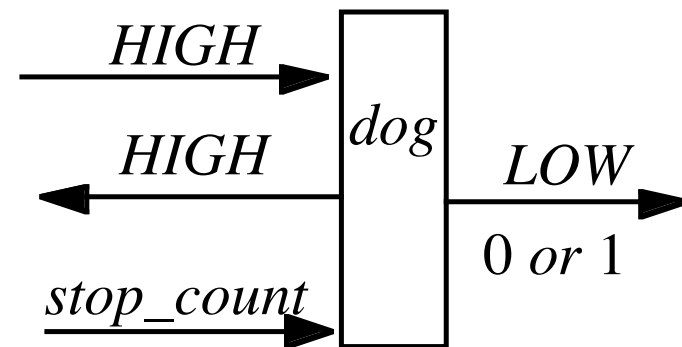
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- If even number of *High* inputs, output could be:
  - 0 (even number of outputs)
  - 1 (odd number of outputs)
- If odd number of *High* inputs, output could be:
  - 0 (odd number of outputs)
  - 1 (even number of outputs)
- High level inputs do not affect output
  - So noninterference-secure

# Second System: *dog*

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- High outputs to left
- Low outputs of 0 or 1 to right
- *stop\_count* input from the left
  - When it arrives, *dog* emits 0 or 1

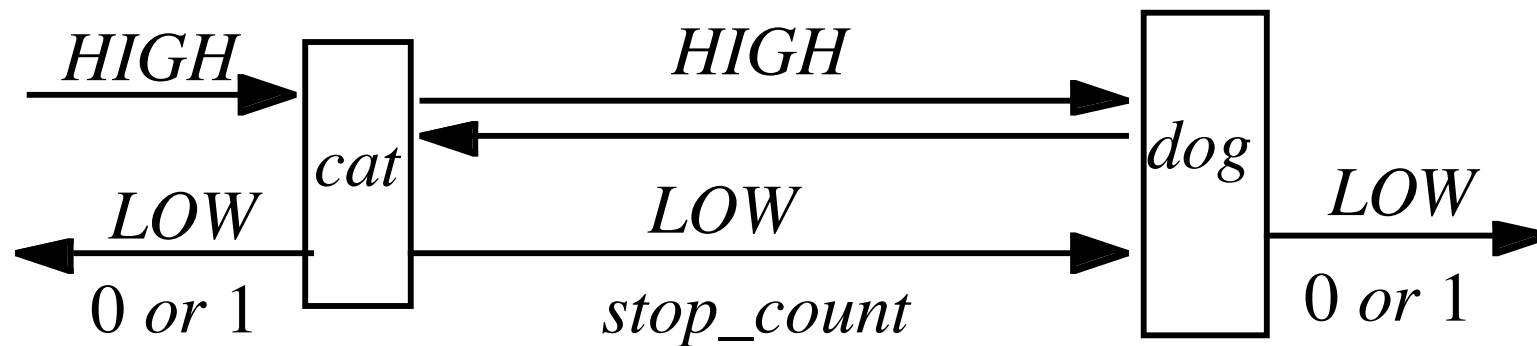


# Noninterference-Secure?

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- When *stop\_count* arrives:
  - May or may not be inputs for which there are no corresponding outputs
  - Parity of *High* inputs, outputs can be odd or even
  - Hence *dog* emits 0 or 1
- High level inputs do not affect low level outputs
  - So noninterference-secure

# Compose Them



- Once sent, message arrives
  - But *stop\_count* may arrive before all inputs have generated corresponding outputs
  - If so, even number of *High* inputs and outputs on *cat*, but odd number on *dog*
- Four cases arise

# The Cases

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- *cat*, odd number of inputs, outputs; *dog*, even number of inputs, odd number of outputs
  - Input message from *cat* not arrived at *dog*, contradicting assumption
- *cat*, even number of inputs, outputs; *dog*, odd number of inputs, even number of outputs
  - Input message from *dog* not arrived at *cat*, contradicting assumption

# The Cases

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- cat, odd number of inputs, outputs; dog, odd number of inputs, even number of outputs
  - dog sent even number of outputs to cat, so cat has had at least one input from left
- cat, even number of inputs, outputs; dog, even number of inputs, odd number of outputs
  - dog sent odd number of outputs to cat, so cat has had at least one input from left

# The Conclusion

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- Composite system *catdog* emits 0 to left, 1 to right (or 1 to left, 0 to right)
  - Must have received at least one input from left
- Composite system *catdog* emits 0 to left, 0 to right (or 1 to left, 1 to right)
  - Could not have received any from left
- So, *High* inputs affect *Low* outputs
  - Not noninterference-secure

# Feedback-Free Systems

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- System has  $n$  distinct components
- Components  $c_i, c_j$  connected if any output of  $c_i$  is input to  $c_j$
- System is *feedback-free* if for all  $c_i$  connected to  $c_j$ ,  $c_j$  not connected to any  $c_i$ 
  - Intuition: once information flows from one component to another, no information flows back from the second to the first



# Feedback-Free Security

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- *Theorem:* A feedback-free system composed of noninterference-secure systems is itself noninterference-secure

# Some Feedback

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- *Lemma:* A noninterference-secure system can feed a high level output  $o$  to a high level input  $i$  if the arrival of  $o$  at the input of the next component is delayed until *after* the next low level input or output
- *Theorem:* A system with feedback as described in the above lemma and composed of noninterference-secure systems is itself noninterference-secure

# Why Didn't They Work?

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- For compositions to work, machine must act same way regardless of what precedes low level input (high, low, nothing)
- *dog* does not meet this criterion
  - If first input is *stop\_count*, *dog* emits 0
  - If high level input precedes *stop\_count*, *dog* emits 0 or 1

# State Machine Model

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- 2-bit machine, levels *High*, *Low*, meeting 4 properties:
  1. For every input  $i_k$ , state  $\sigma_j$ , there is an element  $c_m \in C^*$  such that  $T^*(c_m, \sigma_j) = \sigma_n$ , where  $\sigma_n \neq \sigma_j$ 
    - $T^*$  is total function, inputs and commands always move system to a different state

# Property 2

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- There is an equivalence relation  $\equiv$  such that:
  - If system in state  $\sigma_i$  and high level sequence of inputs causes transition from  $\sigma_i$  to  $\sigma_j$ , then  $\sigma_i \equiv \sigma_j$
  - If  $\sigma_i \equiv \sigma_j$  and low level sequence of inputs  $i_1, \dots, i_n$  causes system in state  $\sigma_i$  to transition to  $\sigma_i'$ , then there is a state  $\sigma_j'$  such that  $\sigma_i' \equiv \sigma_j'$  and the inputs  $i_1, \dots, i_n$  cause system in state  $\sigma_j$  to transition to  $\sigma_j'$
- $\equiv$  holds if low level projections of both states are same

# Property 3

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- Let  $\sigma_i \equiv \sigma_j$ . If high level sequence of outputs  $o_1, \dots, o_n$  indicate system in state  $\sigma_i$  transitioned to state  $\sigma_i'$ , then for some state  $\sigma_j'$  with  $\sigma_j' \equiv \sigma_i'$ , high level sequence of outputs  $o_1', \dots, o_m'$  indicates system in  $\sigma_j$  transitioned to  $\sigma_j'$ 
  - High level outputs do not indicate changes in low level projection of states

# Property 4

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- Let  $\sigma_i \equiv \sigma_j$ , let  $c, d$  be high level output sequences,  $e$  a low level output. If  $ced$  indicates system in state  $\sigma_i$  transitions to  $\sigma_i'$ , then there are high level output sequences  $c'$  and  $d'$  and state  $\sigma_j'$  such that  $c'ed'$  indicates system in state  $\sigma_j$  transitions to state  $\sigma_j'$ 
  - Intermingled low level, high level outputs cause changes in low level state reflecting low level outputs only

# Restrictiveness

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- System is *restrictive* if it meets the preceding 4 properties



# Composition

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- Intuition: by 3 and 4, high level output followed by low level output has same effect as low level input, so composition of restrictive systems should be restrictive

# Composite System

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- System  $M_1$ 's outputs are  $M_2$ 's inputs
- $\mu_{1i}, \mu_{2i}$  states of  $M_1, M_2$
- States of composite system pairs of  $M_1, M_2$  states  $(\mu_{1i}, \mu_{2i})$
- $e$  event causing transition
- $e$  causes transition from state  $(\mu_{1a}, \mu_{2a})$  to state  $(\mu_{1b}, \mu_{2b})$  if any of 3 conditions hold

# Conditions

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1.  $M_1$  in state  $\mu_{1a}$  and  $e$  occurs,  $M_1$  transitions to  $\mu_{1b}$ ;  $e$  not an event for  $M_2$ ; and  $\mu_{2a} = \mu_{2b}$
2.  $M_2$  in state  $\mu_{2a}$  and  $e$  occurs,  $M_2$  transitions to  $\mu_{2b}$ ;  $e$  not an event for  $M_1$ ; and  $\mu_{1a} = \mu_{1b}$
3.  $M_1$  in state  $\mu_{1a}$  and  $e$  occurs,  $M_1$  transitions to  $\mu_{1b}$ ;  $M_2$  in state  $\mu_{2a}$  and  $e$  occurs,  $M_2$  transitions to  $\mu_{2b}$ ;  $e$  is input to one machine, and output from other

# Intuition

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- Event causing transition in composite system causes transition in at least 1 of the components
- If transition occurs in exactly one component, event must not cause transition in other component when not connected to the composite system

# Equivalence for Composite

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- Equivalence relation for composite system  
 $(\sigma_a, \sigma_b) \equiv_C (\sigma_c, \sigma_d)$  iff  $\sigma_a \equiv \sigma_c$  and  $\sigma_b \equiv \sigma_d$
- Corresponds to equivalence relation in property 2 for component system