Lecture 17: Information Flow

- Basics and background
  - Entropy
- Nonlattice flow policies
- Compiler-based mechanisms
- Execution-based mechanisms
- Examples
  - Security Pipeline Interface
  - Secure Network Server Mail Guard

Basics

- Bell-LaPadula Model embodies information flow policy
  - Given compartments A, B, info can flow from A to B iff B *dom* A
- Variables x, y assigned compartments $\underline{x}$, $\underline{y}$ as well as values
  - If $\underline{x} = A$ and $\underline{y} = \mathrm{B}$, and A *dom* B, then $y := x$ allowed but not $x := y$

Quick Review of Entropy

- Random variables
- Joint probability
- Conditional probability
- Entropy (or uncertainty in bits)
- Joint entropy
- Conditional entropy
- Applying it to secrecy of ciphers

Random Variable

- Variable that represents outcome of an event
  - X represents value from roll of a fair die; probability for rolling n: $p(X = n) = 1/6$
  - If die is loaded so 2 appears twice as often as other numbers, $p(X = 2) = 2/7$ and, for $n \neq 2$, $p(X = n) = 1/7$
- Note: $p(X)$ means specific value for X doesn't matter
  - Example: all values of X are equiprobable

Joint Probability

- Joint probability of X and Y, $p(X, Y)$, is probability that X and Y simultaneously assume particular values
  - If X, Y independent, $p(X, Y) = p(X)p(Y)$
- Roll die, toss coin
  - $p(X = 3, Y = \text{heads}) = p(X = 3)p(Y = \text{heads}) = 1/6 \times 1/2 = 1/12$

Two Dependent Events

- X = roll of red die, Y = sum of red, blue die rolls

  $p(Y=2) = 1/36$ $p(Y=3) = 2/36$ $p(Y=4) = 3/36$ $p(Y=5) = 4/36$

  $p(Y=6) = 5/36$ $p(Y=7) = 6/36$ $p(Y=8) = 5/36$ $p(Y=9) = 4/36$

  $p(Y=10) = 3/36$ $p(Y=11) = 2/36$ $p(Y=12) = 1/36$

- Formula:
  - $p(X=1, Y=11) = p(X=1)p(Y=11) = (1/6)(2/36) = 1/108$

Conditional Probability

- Conditional probability of X given Y, $p(X|Y)$, is probability that X takes on a particular value given Y has a particular value
- Continuing example …
  - $p(Y=7|X=1) = 1/6$
  - $p(Y=7|X=3) = 1/6$

Relationship

- $p(X, Y) = p(X \mid Y)\ p(Y) = p(X)\ p(Y \mid X)$
- Example:
  - $p(X=3, Y=8) = p(X=3 \mid Y=8)\ p(Y=8) = (1/5)\ (5/36) = 1/36$
- Note: if X, Y independent:
  - $p(X \mid Y) = p(X)$

Entropy

- Uncertainty of a value, as measured in bits
- Example: X value of fair coin toss; X could be heads or tails, so 1 bit of uncertainty
  - Therefore entropy of X is $H(X) = 1$
- Formal definition: random variable X, values $x_1, \ldots, x_n$; so $\Sigma_i \, \mathrm{p}(X = x_i) = 1$

  $H(X) = -\Sigma_i \, p(X = x_i) \lg p(X = x_i)$

Heads or Tails?

- $H(X) = -p(X=\text{heads}) \lg p(X=\text{heads})$
  $- \text{p}(X=\text{tails}) \lg p(X=\text{tails})$
  $= -(1/2) \lg (1/2) - (1/2) \lg (1/2)$
  $= -(1/2)(-1) - (1/2)(-1) = 1$
- Confirms previous intuitive result

n-Sided Fair Die

$H(X) = -\Sigma_i\, p(X = x_i) \lg p(X = x_i)$

As $p(X = x_i) = 1/n$, this becomes

$H(X) = -\Sigma_i\, (1/n) \lg (1/ n) = -n(1/n)\, (-\lg n)$

so

$H(X) = \lg n$

which is the number of bits in n, as expected

Ann, Pam, and Paul

Ann, Pam twice as likely to win as Paul

W represents the winner. What is its entropy?

- w_1 = Ann, w_2 = Pam, w_3 = Paul
- $p(W = w_1) = p(W = w_2) = 2/5$, $p(W = w_3) = 1/5$

- So $H(W) = -\Sigma_i\, p(W = w_i) \lg p(W = w_i)$

  $= -(2/5) \lg (2/5) - (2/5) \lg (2/5) - (1/5) \lg (1/5)$

  $= -(4/5) + \lg 5 \approx -1.52$
- If all equally likely to win, $H(W) = \lg 3 = 1.58$

Joint Entropy

- X takes values from $\{ x_1, \ldots, x_n \}$
  - $\Sigma_i \, p(X=x_i) = 1$
- Y takes values from $\{ y_1, \ldots, y_m \}$
  - $\Sigma_i \, p(Y=y_i) = 1$
- Joint entropy of X, Y is:
  - $H(X, Y) = -\Sigma_j \, \Sigma_i \, p(X=x_i, Y=y_j) \lg p(X=x_i, Y=y_j)$

Example

X: roll of fair die, Y: flip of coin

$p(X=1, Y=\text{heads}) = p(X=1)\ p(Y=\text{heads}) = 1/12$

– As X and Y are independent

$$H(X, Y) = -\Sigma_j \Sigma_i\ p(X=x_i, Y=y_j) \lg p(X=x_i, Y=y_j)$$
$$= -2\ [\ 6\ [\ (1/12) \lg (1/12)\ ]\ ] = \lg 12$$

Conditional Entropy

- X takes values from $\{ x_1, \ldots, x_n \}$
  - $\Sigma_i \, p(X=x_i) = 1$
- Y takes values from $\{ y_1, \ldots, y_m \}$
  - $\Sigma_i \, p(Y=y_i) = 1$
- Conditional entropy of X given $Y=y_j$ is:
  - $H(X \mid Y=y_j) = -\Sigma_i \, p(X=x_i \mid Y=y_j) \lg p(X=x_i \mid Y=y_j)$
- Conditional entropy of X given Y is:
  - $H(X \mid Y) = -\Sigma_j \, p(Y=y_j) \, \Sigma_i \, p(X=x_i \mid Y=y_j) \lg p(X=x_i \mid Y=y_j)$

Example

- X roll of red die, Y sum of red, blue roll
- Note $p(X=1|Y=2) = 1$, $p(X=i|Y=2) = 0$ for $i \neq 1$
  - If the sum of the rolls is 2, both dice were 1
- $H(X|Y=2) = -\Sigma_i\, p(X=x_i|Y=2) \lg p(X=x_i|Y=2) = 0$
- Note $p(X=\text{i},Y=7) = 1/6$
  - If the sum of the rolls is 7, the red die can be any of 1, …, 6 and the blue die must be 7–roll of red die
- $H(X|Y=7) = -\Sigma_i\, p(X=x_i|Y=7) \lg p(X=x_i|Y=7)$

$$= -6\ (1/6) \lg (1/6) = \lg 6$$

Perfect Secrecy

- Cryptography: knowing the ciphertext does not decrease the uncertainty of the plaintext
- $M = \{ m_1, \ldots, m_n \}$ set of messages
- $C = \{ c_1, \ldots, c_n \}$ set of messages
- Cipher $c_i = E(m_i)$ achieves *perfect secrecy* if $H(M \mid C) = H(M)$

Entropy and Information Flow

- Idea: info flows from x to y as a result of a sequence of commands c if you can deduce information about x before c from the value in y after c
- Formally:
  - s time before execution of c, t time after
  - $H(x_s \mid y_t) < H(x_s \mid y_s)$
  - If no y at time s, then $H(x_s \mid y_t) < H(x_s)$

Example 1

- Command is $x := y + z$; where:
  - $0 \le y \le 7$, equal probability
  - $z = 1$ with prob. 1/2, $z = 2$ or 3 with prob. 1/4 each
- s state before command executed; t, after; so
  - $H(y_s) = H(y_t) = -8(1/8) \lg (1/8) = 3$
  - $H(z_s) = H(z_t) = -(1/2) \lg (1/2) - 2(1/4) \lg (1/4) = 1.5$
- If you know x_t, y_s can have at most 3 values, so $H(y_s \mid x_t) = -3(1/3) \lg (1/3) = \lg 3$

Example 2

- Command is
  - **if** $x = 1$ **then** $y := 0$ **else** $y := 1$;

  where:
  - x, y equally likely to be either 0 or 1
- $H(x_s) = 1$ as x can be either 0 or 1 with equal probability
- $H(x_s \mid y_t) = 0$ as if $y_t = 1$ then $x_s = 0$ and vice versa
  - Thus, $H(x_s \mid y_t) = 0 < 1 = H(x_s)$
- So information flowed from x to y

Implicit Flow of Information

- Information flows from x to y without an *explicit* assignment of the form $y := f(x)$
  - $f(x)$ an arithmetic expression with variable x
- Example from previous slide:
  - **if** $x = 1$ **then** $y := 0$

    else $y := 1$;
- So must look for implicit flows of information to analyze program

Notation

- $\underline{x}$ means class of x
  - In Bell-LaPadula based system, same as "label of security compartment to which x belongs"
- $\underline{x} \leq \underline{y}$ means "information can flow from an element in class of x to an element in class of y
  - Or, "information with a label placing it in class $\underline{x}$ can flow into class $\underline{y}$"

Information Flow Policies

Information flow policies are usually:

- reflexive
  - So information can flow freely among members of a single class
- transitive
  - So if information can flow from class 1 to class 2, and from class 2 to class 3, then information can flow from class 1 to class 3

Non-Transitive Policies

- Betty is a confident of Anne
- Cathy is a confident of Betty
  - With transitivity, information flows from Anne to Betty to Cathy
- Anne confides to Betty she is having an affair with Cathy's spouse
  - Transitivity undesirable in this case, probably

Non-Lattice Transitive Policies

- 2 faculty members co-PIs on a grant
  - Equal authority; neither can overrule the other
- Grad students report to faculty members
- Undergrads report to grad students
- Information flow relation is:
  - Reflexive and transitive
- But some elements (people) have no "least upper bound" element
  - What is it for the faculty members?

Confidentiality Policy Model

- Lattice model fails in previous 2 cases
- Generalize: policy $I = (SC_I, \leq_I, join_I)$:
  - SC_I set of security classes
  - $\leq_I$ ordering relation on elements of SC_I
  - $join_I$ function to combine two elements of SC_I
- Example: Bell-LaPadula Model
  - SC_I set of security compartments
  - $\leq_I$ ordering relation *dom*
  - $join_I$ function *lub*

Confinement Flow Model

- $(I, O, confine, \rightarrow)$
  - $I = (SC_I, \leq_I, join_I)$
  - O set of entities
  - $\rightarrow$: $O \times O$ with $(a, b) \in \rightarrow$ (written $a \rightarrow b$) iff information can flow from a to b
  - for $a \in O$, $confine(a) = (a_L, a_U) \in SC_I \times SC_I$ with $a_L \leq_I a_U$
    - Interpretation: for $a \in O$, if $x \leq_I a_U$, info can flow from x to a, and if $a_L \leq_I x$, info can flow from a to x
    - So a_L lowest classification of info allowed to flow out of a, and a_U highest classification of info allowed to flow into a

Assumptions, *etc*.

- Assumes: object can change security classes
  - So, variable can take on security class of its data
- Object x has security class $\underline{x}$ currently
- Note transitivity *not* required
- If information can flow from a to b, then b dominates a under ordering of policy I:

  $(\forall\ a, b \in O)[\ a \rightarrow b \Rightarrow a_L \leq_I b_U\ ]$

Example 1

- $SC_I = \{ \text{U}, \text{C}, \text{S}, \text{TS} \}$, with $\text{U} \leq_I \text{C}$, $\text{C} \leq_I \text{S}$, and $\text{S} \leq_I \text{TS}$
- $a, b, c \in O$
  - confine(a) = [C, C]
  - confine(b) = [S, S]
  - confine(c) = [TS, TS]
- Secure information flows: $a \rightarrow b$, $a \rightarrow c$, $b \rightarrow c$
  - As $a_L \leq_I b_U$, $a_L \leq_I c_U$, $b_L \leq_I c_U$
  - Transitivity holds

Example 2

- $SC_I, \leq_I$ as in Example 1
- $x, y, z \in O$
  - confine(x) = [C, C]
  - confine(y) = [S, S]
  - confine(z) = [C, TS]
- Secure information flows: $x \rightarrow y$, $x \rightarrow z$, $y \rightarrow z$, $z \rightarrow x$, $z \rightarrow y$
  - As $x_L \leq_I y_U$, $x_L \leq_I z_U$, $y_L \leq_I z_U$, $z_L \leq_I x_U$, $z_L \leq_I y_U$
  - Transitivity does not hold
    - $y \rightarrow z$ and $z \rightarrow x$, but $y \rightarrow z$ is false, because $y_L \leq_I x_U$ is false

Transitive Non-Lattice Policies

- Q = (S_Q, $\leq_Q$) is a *quasi-ordered set* when $\leq_Q$ is transitive and reflexive over S_Q
- How to handle information flow?
  - Define a partially ordered set containing quasi-ordered set
  - Add least upper bound, greatest lower bound to partially ordered set
  - It's a lattice, so apply lattice rules!

In Detail …

- $\forall x \in S_Q$: let $f(x) = \{ y \mid y \in S_Q \wedge y \leq_Q x \}$
  - Define $S_{QP} = \{ f(x) \mid x \in S_Q \}$
  - Define $\leq_{QP} = \{ (x, y) \mid x, y \in S_Q \wedge x \subseteq y \}$
    - S_{QP} partially ordered set under $\leq_{QP}$
    - f preserves order, so $y \leq_Q x$ iff $f(x) \leq_{QP} f(y)$
- Add upper, lower bounds
  - $S_{QP}' = S_{QP} \cup \{ S_Q, \varnothing \}$
  - Upper bound $ub(x, y) = \{ z \mid z \in S_{QP} \wedge x \subseteq z \wedge y \subseteq z \}$
  - Least upper bound $lub(x, y) = \cap ub(x, y)$
    - Lower bound, greatest lower bound defined analogously

And the Policy Is …

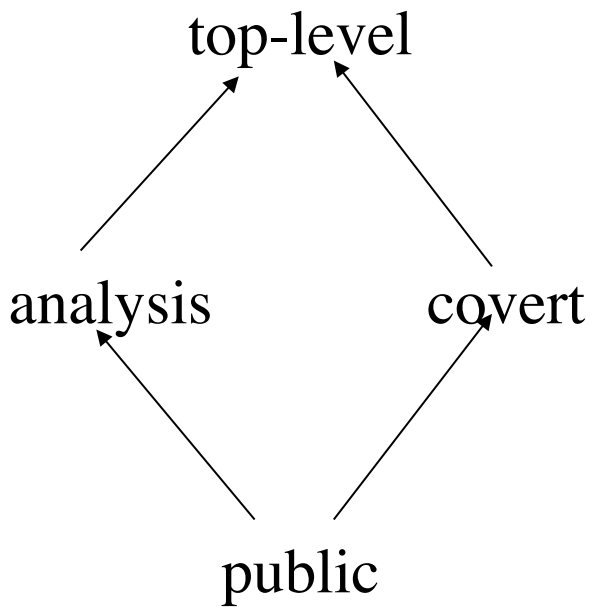
- Now $(S_{QP}', \leq_{QP})$ is lattice
- Information flow policy on quasi-ordered set emulates that of this lattice!

Nontransitive Flow Policies

- Government agency information flow policy (on next slide)
- Entities public relations officers PRO, analysts A, spymasters S
  - *confine*(PRO) = { public, analysis }
  - *confine*(A) = { analysis, top-level }
  - *confine*(S) = { covert, top-level }

Information Flow

- By confinement flow model:
  - PRO ≤ A, A ≤ PRO
  - PRO ≤ S
  - A ≤ S, S ≤ A
- Data *cannot* flow to public relations officers; not transitive
  - S ≤ A, A ≤ PRO
  - S ≤ PRO is *false*



Transforming Into Lattice

- Rough idea: apply a special mapping to generate a subset of the power set of the set of classes
  - Done so this set is partially ordered
  - Means it can be transformed into a lattice
- Can show this mapping preserves ordering relation
  - So it preserves non-orderings and non-transitivity of elements corresponding to those of original set

Dual Mapping

- $R = (SC_R, \leq_R, join_R)$ reflexive info flow policy
- $P = (S_P, \leq_P)$ ordered set
  - Define *dual mapping* functions $l_R, h_R: SC_R \rightarrow S_P$
    - $l_R(x) = \{ x \}$
    - $h_R(x) = \{ y \mid y \in SC_R \wedge y \leq_R x \}$
  - S_P contains subsets of SC_R; $\leq_P$ subset relation
  - Dual mapping function *order preserving* iff

$$(\forall a, b \in SC_R)[\ a \leq_R b \Leftrightarrow l_R(a) \leq_P h_R(b)\ ]$$

Theorem

Dual mapping from reflexive info flow policy R to ordered set P order-preserving

Proof sketch: all notation as before

($\Rightarrow$) Let $a \leq_R b$. Then $a \in l_R(a)$, $a \in h_R(b)$, so $l_R(a) \subseteq h_R(b)$, or $l_R(a) \leq_P h_R(b)$

($\Leftarrow$) Let $l_R(a) \leq_P h_R(b)$. Then $l_R(a) \subseteq h_R(b)$. But $l_R(a) = \{ a \}$, so $a \in h_R(b)$, giving $a \leq_R b$

Info Flow Requirements

- Interpretation: let *confine*(x) = { $\underline{x}_L, \underline{x}_U$ }, consider class $\underline{y}$
  - Information can flow from x to element of $\underline{y}$ iff $\underline{x}_L \leq_R \underline{y}$, or $l_R(\underline{x}_L) \subseteq h_R(\underline{y})$
  - Information can flow from element of $\underline{y}$ to x iff y $\leq_R \underline{x}_U$, or $l_R(\underline{y}) \subseteq h_R(\underline{x}_U)$

Revisit Government Example

- Information flow policy is R
- Flow relationships among classes are:

public $\leq_R$ public

public $\leq_R$ analysis

public $\leq_R$ covert

public $\leq_R$ top-level

analysis $\leq_R$ top-level

analysis $\leq_R$ analysis

covert $\leq_R$ covert

covert $\leq_R$ top-level

top-level $\leq_R$ top-level

Dual Mapping of R

- Elements l_R, h_R:

  l_R(public) = { public }

  h_R(public = { public }

  l_R(analysis) = { analysis }

  h_R(analysis) = { public, analysis }

  l_R(covert) = { covert }

  h_R(covert) = { public, covert }

  l_R(top-level) = { top-level }

  h_R(top-level) = { public, analysis, covert, top-level }

confine

- Let *p* be entity of type PRO, *a* of type A, *s* of type S
- In terms of *P* (not *R*), we get:
  - *confine*(*p*) = [{ public }, { public, analysis }]
  - *confine*(*a*) = [{ analysis },
    { public, analysis, covert, top-level }]
  - *confine*(*s*) = [{ covert },
    { public, analysis, covert, top-level }]

And the Flow Relations Are …

- $p \rightarrow a$ as $l_R(p) \subseteq h_R(a)$
  - $l_R(p)$ = { public }
  - $h_R(a)$ = { public, analysis, covert, top-level }
- Similarly: $a \rightarrow p$, $p \rightarrow s$, $a \rightarrow s$, $s \rightarrow a$
- ***But*** $s \rightarrow p$ ***is false*** as $l_R(s) \not\subseteq h_R(p)$
  - $l_R(s)$ = { covert }
  - $h_R(p)$ = { public, analysis }

Analysis

- $(S_P, \leq_P)$ is a lattice, so it can be analyzed like a lattice policy
- Dual mapping preserves ordering, hence non-ordering and non-transitivity, of original policy
  - So results of analysis of $(S_P, \leq_P)$ can be mapped back into $(SC_R, \leq_R, join_R)$