# ECS 235B Module 40 Restrictiveness

## Feedback-Free Systems

- System has *n* distinct components
- Components  $c_i$ ,  $c_j$  are connected if any output of  $c_i$  is input to  $c_j$
- System is *feedback-free* if for all  $c_i$  connected to  $c_i$ ,  $c_j$  not connected to any  $c_i$ 
  - Intuition: once information flows from one component to another, no information flows back from the second to the first

Feedback-Free Security

• *Theorem*: A feedback-free system composed of noninterference-secure systems is itself noninterference-secure

## Some Feedback

- Lemma: A noninterference-secure system can feed a HIGH output o to a HIGH input i if the arrival of o at the input of the next component is delayed until after the next LOW input or output
- *Theorem*: A system with feedback as described in the above lemma and composed of noninterference-secure systems is itself noninterference-secure

# Why Didn't They Work?

- For compositions to work, machine must act same way regardless of what precedes LOW input (HIGH, LOW, nothing)
- *dog* does not meet this criterion
  - If first input is *stop\_count, dog* emits 0
  - If high level input precedes *stop\_count, dog* emits 0 or 1

## State Machine Model: 2-Bit Machine

Levels *High*, *Low*, meet 4 properties:

1. For every input  $i_k$ , state  $\sigma_j$ , there is an element  $c_m \in C^*$  such that  $T^*(c_m, \sigma_j) = \sigma_n$ , where  $\sigma_n \neq \sigma_j$ 

T\* is total function, inputs and commands always move system to a different state

# Property 2

- 2. There is an equivalence relation  $\equiv$  such that:
  - a. If system in state  $\sigma_i$  and HIGH sequence of inputs causes transition from  $\sigma_i$  to  $\sigma_j$ , then  $\sigma_i \equiv \sigma_j$ 
    - 2 states equivalent if either reachable from the other state using only HIGH commands
  - b. If  $\sigma_i \equiv \sigma_j$  and LOW sequence of inputs  $i_1, ..., i_n$  causes system in state  $\sigma_i$  to transition to  $\sigma'_i$ , then there is a state  $\sigma'_j$  such that  $\sigma'_i \equiv \sigma'_j$  and inputs  $i_1, ..., i_n$  cause system in state  $\sigma_j$  to transition to  $\sigma'_j$ 
    - States resulting from giving same LOW commands to the two equivalent original states have same LOW projection
- $\equiv$  holds if LOW projections of both states are same
  - If 2 states equivalent, HIGH commands do not affect LOW projections

## Property 3

- Let  $\sigma_i \equiv \sigma_j$ . If sequence of HIGH outputs  $o_1, ..., o_n$  indicate system in state  $\sigma_i$  transitioned to state  $\sigma_i'$ , then for some state  $\sigma_j'$  with  $\sigma_j' \equiv \sigma_i'$ , sequence of HIGH outputs  $o_1', ..., o_m'$  indicates system in  $\sigma_j$  transitioned to  $\sigma_j'$ 
  - HIGH outputs do not indicate changes in LOW projection of states

## Property 4

- Let  $\sigma_i \equiv \sigma_j$ , let *c*, *d* be HIGH output sequences, *e* a LOW output. If output sequence *ced* indicates system in state  $\sigma_i$  transitions to  $\sigma_i'$ , then there are HIGH output sequences *c'* and *d'* and state  $\sigma_j'$  such that *c'ed'* indicates system in state  $\sigma_i$  transitions to state  $\sigma_j'$ 
  - Intermingled LOW, HIGH outputs cause changes in LOW state reflecting LOW outputs only

#### Restrictiveness

• System is *restrictive* if it meets the preceding 4 properties

## Composition

 Intuition: by 3 and 4, HIGH output followed by LOW output has same effect as the LOW input, so composition of restrictive systems should be restrictive

## Composite System

- System  $M_1$ 's outputs are acceptable as  $M_2$ 's inputs
- $\mu_{1i}$ ,  $\mu_{2i}$  states of  $M_1$ ,  $M_2$
- States of composite system pairs of  $M_1$ ,  $M_2$  states ( $\mu_{1i}$ ,  $\mu_{2i}$ )
- e event causing transition
- *e* causes transition from state ( $\mu_{1a}$ ,  $\mu_{2a}$ ) to state ( $\mu_{1b}$ ,  $\mu_{2b}$ ) if any of 3 conditions hold

## Conditions

- 1.  $M_1$  in state  $\mu_{1a}$  and *e* occurs,  $M_1$  transitions to  $\mu_{1b}$ ; *e* not an event for  $M_2$ ; and  $\mu_{2a} = \mu_{2b}$
- 2.  $M_2$  in state  $\mu_{2a}$  and *e* occurs,  $M_2$  transitions to  $\mu_{2b}$ ; *e* not an event for  $M_1$ ; and  $\mu_{1a} = \mu_{1b}$
- 3.  $M_1$  in state  $\mu_{1a}$  and e occurs,  $M_1$  transitions to  $\mu_{1b}$ ;  $M_2$  in state  $\mu_{2a}$  and e occurs,  $M_2$  transitions to  $\mu_{2b}$ ; e is input to one machine, and output from other

## Intuition

- Event causing transition in composite system causes transition in at least 1 of the components
- If transition occurs in exactly 1 component, event must not cause transition in other component when not connected to the composite system

# Equivalence for Composite

• Equivalence relation for composite system

$$(\sigma_a, \sigma_b) \equiv_C (\sigma_c, \sigma_d) \text{ iff } \sigma_a \equiv \sigma_c \text{ and } \sigma_b \equiv \sigma_d$$

 Corresponds to equivalence relation in property 2 for component system



The system resulting from the composition of two restrictive systems is itself restrictive

## Quiz

# True or False: The composition of two restrictive systems is not restrictive