ECS 235B Module 11
Expressiveness

Expressive Power

- How do the sets of systems that models can describe compare?
  - If HRU equivalent to SPM, SPM provides more specific answer to safety question
  - If HRU describes more systems, SPM applies only to the systems it can describe

HRU *vs*. SPM

- SPM more abstract
  - Analyses focus on limits of model, not details of representation
- HRU allows revocation
  - SMP has no equivalent to delete, destroy
- HRU allows multiparent creates
  - SMP cannot express multiparent creates easily, and not at all if the parents are of different types because *can•create* allows for only one type of creator

Multiparent Create

- Solves mutual suspicion problem
  - Create proxy jointly, each gives it needed rights
- In HRU:

command *multicreate*(s_0, s_1, o)
if r **in** $a[s_0, s_1]$ **and** r **in** $a[s_1, s_0]$
then
  create object o;
  enter r **into** $a[s_0, o]$;
  enter r **into** $a[s_1, o]$;
end

SPM and Multiparent Create

- *cc* extended in obvious way
  - $cc \subseteq TS \times \ldots \times TS \times T$
- Symbols
  - $\mathbf{X}_1, \ldots, \mathbf{X}_n$ parents, $\mathbf{Y}$ created
  - $R_{1,i}, R_{2,i}, R_3, R_{4,i} \subseteq R$
- Rules
  - $cr_{\mathrm{P},i}(\tau(\mathbf{X}_1), \ldots, \tau(\mathbf{X}_n)) = \mathbf{Y}/R_{1,1} \cup \mathbf{X}_i/R_{2,i}$
  - $cr_{\mathrm{C}}(\tau(\mathbf{X}_1), \ldots, \tau(\mathbf{X}_n)) = \mathbf{Y}/R_3 \cup \mathbf{X}_1/R_{4,1} \cup \ldots \cup \mathbf{X}_n/R_{4,n}$

Example

- Anna, Bill must do something cooperatively
  - But they don't trust each other
- Jointly create a proxy
  - Each gives proxy only necessary rights
- In ESPM:
  - Anna, Bill type a; proxy type p; right $x \in R$
  - $cc(a, a) = p$
  - $cr_{\text{Anna}}(a, a, p) = cr_{\text{Bill}}(a, a, p) = \varnothing$
  - $cr_{\text{proxy}}(a, a, p) = \{ \text{Anna}/x, \text{Bill}/x \}$

2-Parent Joint Create Suffices

- Goal: emulate 3-parent joint create with 2-parent joint create
- Definition of 3-parent joint create (subjects $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$; child $\mathbf{C}$):
  - $cc(\tau(\mathbf{P}_1), \tau(\mathbf{P}_2), \tau(\mathbf{P}_3)) = Z \subseteq T$
  - $cr_{\mathbf{P}1}(\tau(\mathbf{P}_1), \tau(\mathbf{P}_2), \tau(\mathbf{P}_3)) = \mathbf{C}/R_{1,1} \cup \mathbf{P}_1/R_{2,1}$
  - $cr_{\mathbf{P}2}(\tau(\mathbf{P}_1), \tau(\mathbf{P}_2), \tau(\mathbf{P}_3)) = \mathbf{C}/R_{2,1} \cup \mathbf{P}_2/R_{2,2}$
  - $cr_{\mathbf{P}3}(\tau(\mathbf{P}_1), \tau(\mathbf{P}_2), \tau(\mathbf{P}_3)) = \mathbf{C}/R_{3,1} \cup \mathbf{P}_3/R_{2,3}$

General Approach

- Define agents for parents and child
  - Agents act as surrogates for parents
  - If create fails, parents have no extra rights
  - If create succeeds, parents, child have exactly same rights as in 3-parent creates
    - Only extra rights are to agents (which are never used again, and so these rights are irrelevant)

Entities and Types

- Parents $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$ have types p_1, p_2, p_3
- Child **C** of type c
- Parent agents $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$ of types a_1, a_2, a_3
- Child agent **S** of type s
- Type t is parentage
  - if $\mathbf{X}/t \in dom(\mathbf{Y})$, **X** is **Y**'s parent
- Types t, a_1, a_2, a_3, s are new types

can•create

- Following added to *can•create*:
  - cc(p_1) = a_1
  - cc(p_2, a_1) = a_2
  - cc(p_3, a_2) = a_3
    - Parents creating their agents; note agents have maximum of 2 parents
  - cc(a_3) = s
    - Agent of all parents creates agent of child
  - cc(s) = c
    - Agent of child creates child

Creation Rules

- Following added to create rule:
  - $cr_P(p_1, a_1) = \varnothing$
  - $cr_C(p_1, a_1) = p_1/Rtc$
    - Agent's parent set to creating parent; agent has all rights over parent
  - $cr_{Pfirst}(p_2, a_1, a_2) = \varnothing$
  - $cr_{Psecond}(p_2, a_1, a_2) = \varnothing$
  - $cr_C(p_2, a_1, a_2) = p_2/Rtc \cup a_1/tc$
    - Agent's parent set to creating parent and agent; agent has all rights over parent (but not over agent)

Creation Rules

- $cr_{Pfirst}(p_3, a_2, a_3) = \varnothing$
- $cr_{Psecond}(p_3, a_2, a_3) = \varnothing$
- $cr_C(p_3, a_2, a_3) = p_3/Rtc \cup a_2/tc$
  - Agent's parent set to creating parent and agent; agent has all rights over parent (but not over agent)
- $cr_P(a_3, s) = \varnothing$
- $cr_C(a_3, s) = a_3/tc$
  - Child's agent has third agent as parent $cr_P(a_3, s) = \varnothing$
- $cr_P(s, c) = \mathbf{C}/Rtc$
- $cr_C(s, c) = c/R_3t$
  - Child's agent gets full rights over child; child gets R_3 rights over agent

Link Predicates

- Idea: no tickets to parents until child created
  - Done by requiring each agent to have its own parent rights
  - $link_1(\mathbf{A}_2, \mathbf{A}_1) = \mathbf{A}_1/t \in dom(\mathbf{A}_2) \land \mathbf{A}_2/t \in dom(\mathbf{A}_2)$
  - $link_1(\mathbf{A}_3, \mathbf{A}_2) = \mathbf{A}_2/t \in dom(\mathbf{A}_3) \land \mathbf{A}_3/t \in dom(\mathbf{A}_3)$
  - $link_2(\mathbf{S}, \mathbf{A}_3) = \mathbf{A}_3/t \in dom(\mathbf{S}) \land \mathbf{C}/t \in dom(\mathbf{C})$
  - $link_3(\mathbf{A}_1, \mathbf{C}) = \mathbf{C}/t \in dom(\mathbf{A}_1)$
  - $link_3(\mathbf{A}_2, \mathbf{C}) = \mathbf{C}/t \in dom(\mathbf{A}_2)$
  - $link_3(\mathbf{A}_3, \mathbf{C}) = \mathbf{C}/t \in dom(\mathbf{A}_3)$
  - $link_4(\mathbf{A}_1, \mathbf{P}_1) = \mathbf{P}_1/t \in dom(\mathbf{A}_1) \land \mathbf{A}_1/t \in dom(\mathbf{A}_1)$
  - $link_4(\mathbf{A}_2, \mathbf{P}_2) = \mathbf{P}_2/t \in dom(\mathbf{A}_2) \land \mathbf{A}_2/t \in dom(\mathbf{A}_2)$
  - $link_4(\mathbf{A}_3, \mathbf{P}_3) = \mathbf{P}_3/t \in dom(\mathbf{A}_3) \land \mathbf{A}_3/t \in dom(\mathbf{A}_3)$

Filter Functions

- $f_1(a_2, a_1) = a_1/t \cup c/Rtc$
- $f_1(a_3, a_2) = a_2/t \cup c/Rtc$
- $f_2(s, a_3) = a_3/t \cup c/Rtc$
- $f_3(a_1, c) = p_1/R_{4,1}$
- $f_3(a_2, c) = p_2/R_{4,2}$
- $f_3(a_3, c) = p_3/R_{4,3}$
- $f_4(a_1, p_1) = c/R_{1,1} \cup p_1/R_{2,1}$
- $f_4(a_2, p_2) = c/R_{1,2} \cup p_2/R_{2,2}$
- $f_4(a_3, p_3) = c/R_{1,3} \cup p_3/R_{2,3}$

Construction

Create $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, $\mathbf{S}$, $\mathbf{C}$; then

- $\mathbf{P}_1$ has no relevant tickets
- $\mathbf{P}_2$ has no relevant tickets
- $\mathbf{P}_3$ has no relevant tickets
- $\mathbf{A}_1$ has $\mathbf{P}_1/Rtc$
- $\mathbf{A}_2$ has $\mathbf{P}_2/Rtc \cup \mathbf{A}_1/tc$
- $\mathbf{A}_3$ has $\mathbf{P}_3/Rtc \cup \mathbf{A}_2/tc$
- $\mathbf{S}$ has $\mathbf{A}_3/tc \cup \mathbf{C}/Rtc$
- $\mathbf{C}$ has $\mathbf{C}/R_3t$

Construction

- Only $link_2(\mathbf{S}, \mathbf{A}_3)$ true $\Rightarrow$ apply f_2
  - $\mathbf{A}_3$ has $\mathbf{P}_3/Rtc \cup \mathbf{A}_2/t \cup \mathbf{A}_3/t \cup \mathbf{C}/Rtc$
- Now $link_1(\mathbf{A}_3, \mathbf{A}_2)$ true $\Rightarrow$ apply f_1
  - $\mathbf{A}_2$ has $\mathbf{P}_2/Rtc \cup \mathbf{A}_1/tc \cup \mathbf{A}_2/t \cup \mathbf{C}/Rtc$
- Now $link_1(\mathbf{A}_2, \mathbf{A}_1)$ true $\Rightarrow$ apply f_1
  - $\mathbf{A}_1$ has $\mathbf{P}_2/Rtc \cup \mathbf{A}_1/t \cup \mathbf{C}/Rtc$
- Now all $link_3$s true $\Rightarrow$ apply f_3
  - $\mathbf{C}$ has $\mathbf{C}/R_3 \cup \mathbf{P}_1/R_{4,1} \cup \mathbf{P}_2/R_{4,2} \cup \mathbf{P}_3/R_{4,3}$

Finish Construction

- Now $link_4$ is true $\Rightarrow$ apply f_4
  - $\mathbf{P}_1$ has $\mathbf{C}/R_{1,1} \cup \mathbf{P}_1/R_{2,1}$
  - $\mathbf{P}_2$ has $\mathbf{C}/R_{1,2} \cup \mathbf{P}_2/R_{2,2}$
  - $\mathbf{P}_3$ has $\mathbf{C}/R_{1,3} \cup \mathbf{P}_3/R_{2,3}$
- 3-parent joint create gives same rights to $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{C}$
- If create of **C** fails, $link_2$ fails, so construction fails

Theorem

- The two-parent joint creation operation can implement an *n*-parent joint creation operation with a fixed number of additional types and rights, and augmentations to the link predicates and filter functions.
- **Proof**: by construction, as above
  - Difference is that the two systems need not start at the same initial state

Theorems

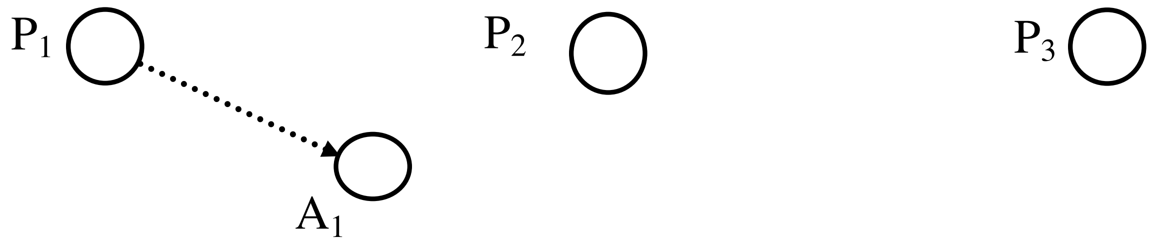
- Monotonic ESPM and the monotonic HRU model are equivalent.
- Safety question in ESPM also decidable if acyclic attenuating scheme
  - Proof similar to that for SPM

Expressiveness

- Graph-based representation to compare models
- Graph
  - Vertex: represents entity, has static type
  - Edge: represents right, has static type
- Graph rewriting rules:
  - *Initial state operations* create graph in a particular state
  - *Node creation operations* add nodes, incoming edges
  - *Edge adding operations* add new edges between existing vertices

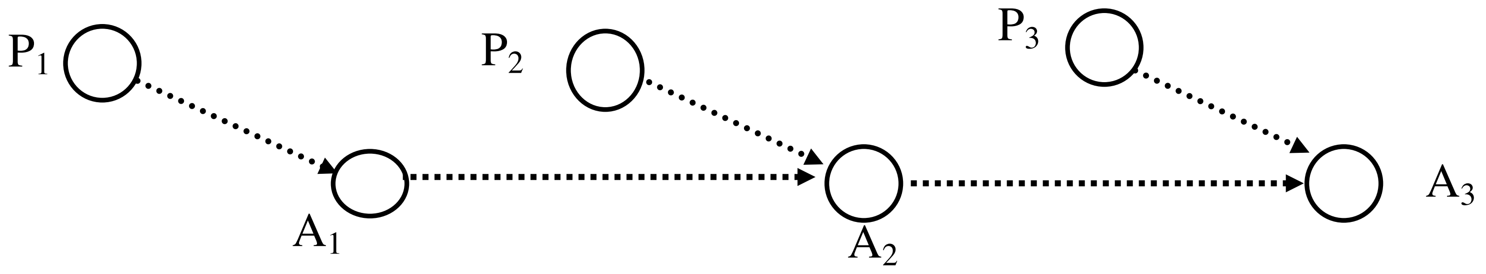
Example: 3-Parent Joint Creation

- Simulate with 2-parent
  - Nodes $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$ parents
  - Create node **C** with type *c* with edges of type *e*
  - Add node $\mathbf{A}_1$ of type *a* and edge from $\mathbf{P}_1$ to $\mathbf{A}_1$ of type *e´*



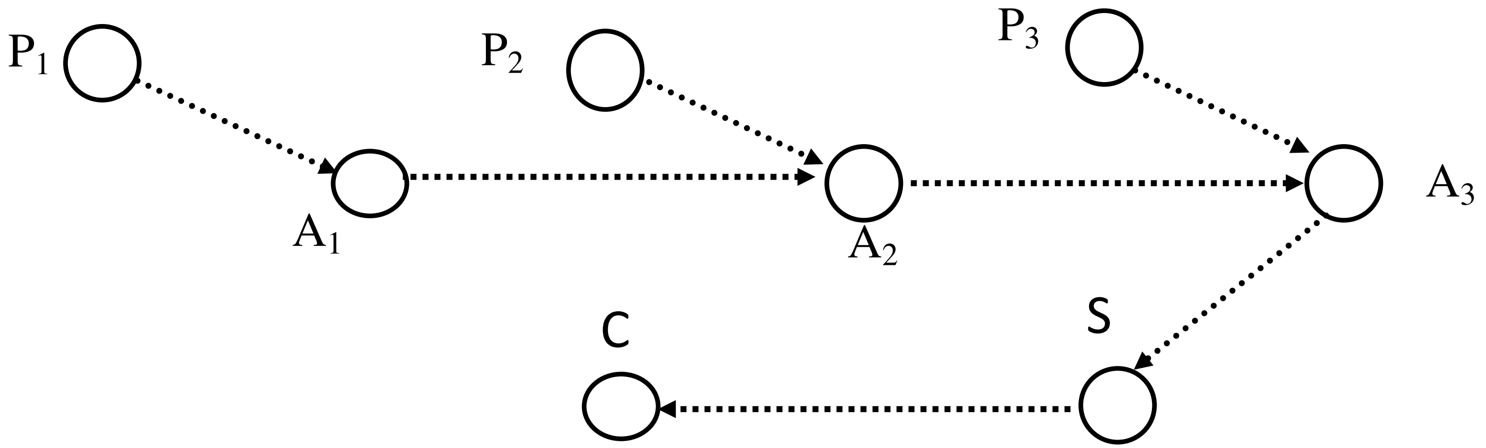
Next Step

- $\mathbf{A}_1$, $\mathbf{P}_2$ create $\mathbf{A}_2$; $\mathbf{A}_2$, $\mathbf{P}_3$ create $\mathbf{A}_3$
- Type of nodes, edges are *a* and *e´*



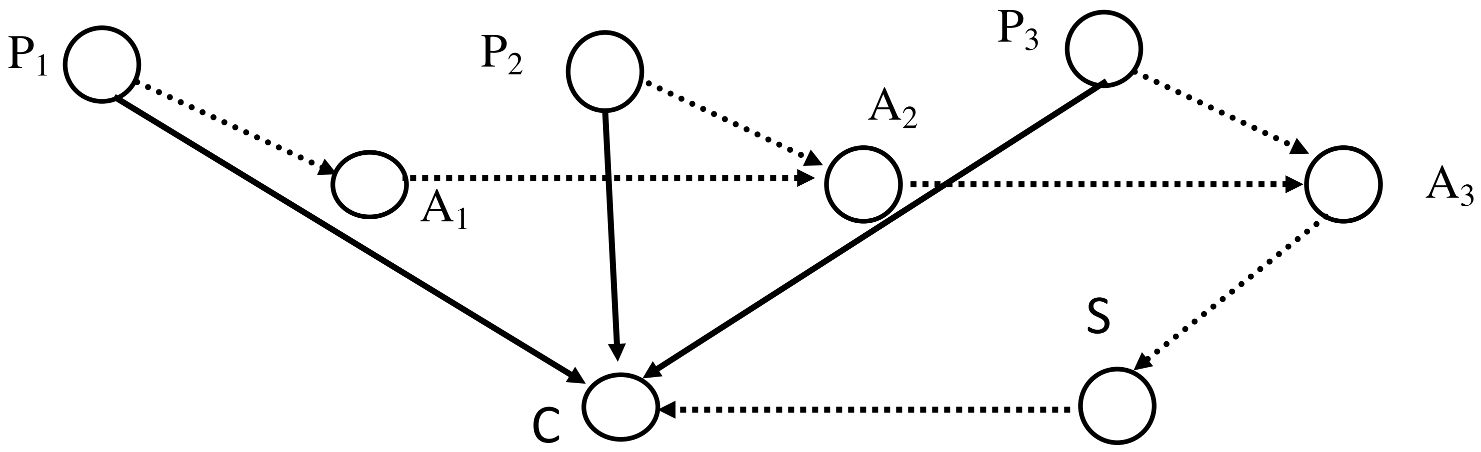
Next Step

- $\mathbf{A}_3$ creates **S**, of type *a*
- **S** creates **C**, of type *c*



Last Step

- Edge adding operations:
  - $\mathbf{P}_1 \to \mathbf{A}_1 \to \mathbf{A}_2 \to \mathbf{A}_3 \to \mathbf{S} \to \mathbf{C}$: $\mathbf{P}_1$ to **C** edge type *e*
  - $\mathbf{P}_2 \to \mathbf{A}_2 \to \mathbf{A}_3 \to \mathbf{S} \to \mathbf{C}$: $\mathbf{P}_2$ to **C** edge type *e*
  - $\mathbf{P}_3 \to \mathbf{A}_3 \to \mathbf{S} \to \mathbf{C}$: $\mathbf{P}_3$ to **C** edge type *e*



Definitions

- *Scheme*: graph representation as above
- *Model*: set of schemes
- Schemes *A*, *B correspond* if graph for both is identical when all nodes with types not in *A* and edges with types in *A* are deleted

Example

- Above 2-parent joint creation simulation in scheme *TWO*
- Equivalent to 3-parent joint creation scheme *THREE* in which $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, **C** are of same type as in *TWO*, and edges from $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$ to **C** are of type *e*, and no types *a* and *e*´ exist in *TWO*

Simulation

Scheme *A* simulates scheme *B* iff

- every state *B* can reach has a corresponding state in *A* that *A* can reach; and
- every state that *A* can reach either corresponds to a state *B* can reach, or has a successor state that corresponds to a state *B* can reach
  - The last means that *A* can have intermediate states not corresponding to states in *B*, like the intermediate ones in *TWO* in the simulation of *THREE*

Expressive Power

- If there is a scheme in *MA* that no scheme in *MB* can simulate, *MB less expressive than MA*
- If every scheme in *MA* can be simulated by a scheme in *MB*, *MB as expressive as MA*
- If *MA* as expressive as *MB* and *vice versa*, *MA* and *MB equivalent*

Example

- Scheme *A* in model *M*
  - Nodes $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$
  - 2-parent joint create
  - 1 node type, 1 edge type
  - No edge adding operations
  - Initial state: $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$, no edges
- Scheme *B* in model *N*
  - All same as *A* except no 2-parent joint create
  - 1-parent create
- Which is more expressive?

Can *A* Simulate *B*?

- Scheme *A* simulates 1-parent create: have both parents be same node
  - Model *M* as expressive as model *N*

Can *B* Simulate *A*?

- Suppose $\mathbf{X}_1$, $\mathbf{X}_2$ jointly create **Y** in *A*
  - Edges from $\mathbf{X}_1$, $\mathbf{X}_2$ to **Y**, no edge from $\mathbf{X}_3$ to **Y**
- Can *B* simulate this?
  - Without loss of generality, $\mathbf{X}_1$ creates **Y**
  - Must have edge adding operation to add edge from $\mathbf{X}_2$ to **Y**
  - One type of node, one type of edge, so operation can add edge between any 2 nodes

No

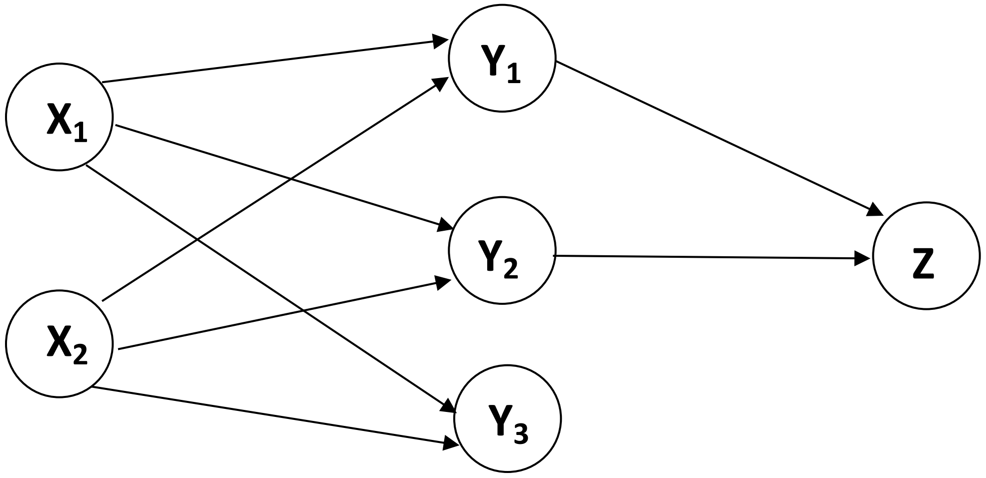
- All nodes in *A* have even number of incoming edges
  - 2-parent create adds 2 incoming edges
- Edge adding operation in *B* that can edge from $\mathbf{X}_2$ to **C** can add one from $\mathbf{X}_3$ to **C**
  - *A* cannot enter this state
  - *B* cannot transition to a state in which **Y** has even number of incoming edges
    - No remove rule
- So *B* cannot simulate *A*; *N* less expressive than *M*

Theorem

- Monotonic single-parent models are less expressive than monotonic multiparent models
- Proof by contradiction
  - Scheme *A* is multiparent model
  - Scheme *B* is single parent create
  - Claim: *B* can simulate *A*, without assumption that they start in the same initial state
    - Note: example assumed same initial state

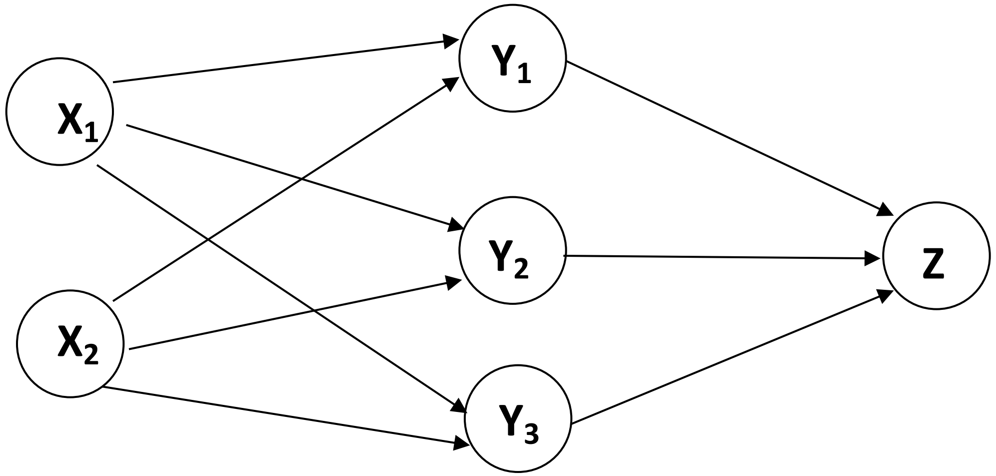
Outline of Proof

- $\mathbf{X}_1$, $\mathbf{X}_2$ nodes in *A*
  - They create $\mathbf{Y}_1$, $\mathbf{Y}_2$, $\mathbf{Y}_3$ using multiparent create rule
  - $\mathbf{Y}_1$, $\mathbf{Y}_2$ create **Z**, again using multiparent create rule
  - *Note*: no edge from $\mathbf{Y}_3$ to **Z** can be added, as *A* has no edge-adding operation



Outline of Proof

- **W**, $\mathbf{X}_1$, $\mathbf{X}_2$ nodes in *B*
  - **W** creates $\mathbf{Y}_1$, $\mathbf{Y}_2$, $\mathbf{Y}_3$ using single parent create rule, and adds edges for $\mathbf{X}_1$, $\mathbf{X}_2$ to all using edge adding rule
  - $\mathbf{Y}_1$ creates **Z**, again using single parent create rule; now must add edge from $\mathbf{Y}_2$ to **Z** to simulate *A*
  - Use same edge adding rule to add edge from $\mathbf{Y}_3$ to **Z**: cannot duplicate this in scheme *A*!



Meaning

- Scheme *B* cannot simulate scheme *A*, contradicting hypothesis
- ESPM more expressive than SPM
  - ESPM multiparent and monotonic
  - SPM monotonic but single parent