ECS 235B Module 36
Security Policy and the Unwinding Theorem

Security Policy

- Partitions systems into authorized, unauthorized states
- Authorized states have no forbidden interferences
- Hence a *security policy* is a set of noninterference assertions
    - See previous definition

Alternative Development

- System *X* is a set of protection domains $D = \{ d_1, \ldots, d_n \}$
- When command *c* executed, it is executed in protection domain *dom*(*c*)
- Give alternate versions of definitions shown previously

Security Policy

- $D = \{ d_1, \ldots, d_n \}$, d_i a protection domain
- r: $D \times D$ a reflexive relation
- Then r defines a security policy
- Intuition: defines how information can flow around a system
  - $d_i r d_j$ means info can flow from d_i to d_j
  - $d_i r d_i$ as info can flow within a domain

Projection Function

- π' analogue of π, earlier
- Commands, subjects absorbed into protection domains
- $d \in D$, $c \in C$, $c_s \in C^*$
- $\pi'_d(\nu) = \nu$
- $\pi'_d(c_s c) = \pi'_d(c_s)c$ if $dom(c) r d$
- $\pi'_d(c_s c) = \pi'_d(c_s)$ otherwise
- Intuition: if executing *c* interferes with *d*, then *c* is visible; otherwise, as if *c* never executed

Noninterference-Secure

- System has set of protection domains *D*
- System is *noninterference-secure with respect to policy r* if

$$P^*(c, T^*(c_s, \sigma_0)) = P^*(c, T^*(\pi'_d(c_s), \sigma_0))$$

- Intuition: if executing c_s causes the same transitions for subjects in domain *d* as does its projection with respect to domain *d*, then no information flows in violation of the policy

Output-Consistency

- $c \in C$, $dom(c) \in D$
- $\sim^{dom(c)}$ equivalence relation on states of system X
- $\sim^{dom(c)}$ *output-consistent* if

$$\sigma_a \sim^{dom(c)} \sigma_b \Rightarrow P(c, \sigma_a) = P(c, \sigma_b)$$

- Intuition: states are output-consistent if for subjects in *dom*(*c*), projections of outputs for both states after *c* are the same

Lemma

- Let $T^*(c_s, \sigma_0) \sim^d T^*(\pi'_d(c_s), \sigma_0)$ for $c \in C$
- If $\sim^{\mathrm{d}}$ output-consistent, then system is noninterference-secure with respect to policy *r*

Proof

- $d = dom(c)$ for $c \in C$
- By definition of output-consistent,

$$T^*(c_s, \sigma_0) \sim^d T^*(\pi'_d(c_s), \sigma_0)$$

implies

$$P^*(c, T^*(c_s, \sigma_0)) = P^*(c, T^*(\pi'_d(c_s), \sigma_0))$$

- This is definition of noninterference-secure with respect to policy r

Unwinding Theorem

- Links security of sequences of state transition commands to security of individual state transition commands
- Allows you to show a system design is multilevel-secure by showing it matches specs from which certain lemmata derived
  - Says *nothing* about security of system, because of implementation, operation, *etc*. issues

Locally Respects

- *r* is a policy
- System *X* *locally respects* *r* if *dom*(*c*) being noninterfering with $d \in D$ implies $\sigma_a \sim^d T(c, \sigma_a)$
- Intuition: when *X* locally respects *r*, applying *c* under policy *r* to system *X* has no effect on domain *d*

Transition-Consistent

- r policy, $d \in D$
- If $\sigma_a \sim^d \sigma_b$ implies $T(c, \sigma_a) \sim^d T(c, \sigma_b)$, system X is *transition-consistent* under r
- Intuition: command c does not affect equivalence of states under policy r

Theorem

- *r* policy, *X* system that is output consistent, transition consistent, and locally respects *r*
- Then *X* noninterference-secure with respect to policy *r*
- Significance: basis for analyzing systems claiming to enforce noninterference policy
    - Establish conditions of theorem for particular set of commands, states with respect to some policy, set of protection domains
    - Noninterference security with respect to *r* follows

Proof

Must show $\sigma_a \sim^d \sigma_b \Rightarrow T^*(c_s, \sigma_a) \sim^d T^*(\pi'_d(c_s), \sigma_b)$

- Induct on length of c_s
- Basis: if $c_s = \nu$, then $T^*(c_s, \sigma_a) = \sigma_a$ and $\pi'_d(\nu) = \nu$; claim holds
- Hypothesis: for $c_s = c_1 \ldots c_n$, $\sigma_a \sim^d \sigma_b \Rightarrow T^*(c_s, \sigma_a) \sim^d T^*(\pi'_d(c_s), \sigma_b)$

Induction Step

- Consider $c_s c_{n+1}$. Assume $\sigma_a \sim^d \sigma_b$ and look at $T^*(\pi'_d(c_s c_{n+1}), \sigma_b)$
- 2 cases:
  - $dom(c_{n+1})rd$ holds
  - $dom(c_{n+1})rd$ does not hold

$dom(c_{n+1}) rd$ Holds

$T^*(\pi'_d(c_s c_{n+1}), \sigma_b) = T^*(\pi'_d(c_s) c_{n+1}, \sigma_b) = T(c_{n+1}, T^*(\pi'_d(c_s), \sigma_b))$

- By definition of T^* and π'_d

$\sigma_a \sim^d \sigma_b \Rightarrow T(c_{n+1}, \sigma_a) \sim^d T(c_{n+1}, \sigma_b)$

- As X transition-consistent

$T(c_{n+1}, T^*(c_s, \sigma_a)) \sim^d T(c_{n+1}, T^*(\pi'_d(c_s), \sigma_b))$

- By transition-consistency and IH

$T(c_{n+1}, T^*(c_s, \sigma_a)) \sim^d T^*(\pi'_d(c_s c_{n+1}), \sigma_b)$

- By substitution from earlier equality

$T^*(c_s c_{n+1}, \sigma_a) \sim^d T^*(\pi'_d(c_s c_{n+1}), \sigma_b)$

- By definition of T^*

proving hypothesis

$dom(c_{n+1}) rd$ Does Not Hold

$T^*(\pi'_d(c_s c_{n+1}), \sigma_b) = T^*(\pi'_d(c_s), \sigma_b)$

- By definition of π'_d

$T^*(c_s, \sigma_a) = T^*(\pi'_d(c_s c_{n+1}), \sigma_b)$

- By above and IH

$T(c_{n+1}, T^*(c_s, \sigma_a)) \sim^d T^*(c_s, \sigma_a)$

- As X locally respects r, $\sigma \sim^d T(c_{n+1}, \sigma)$ for any σ

$T(c_{n+1}, T^*(c_s, \sigma_a)) \sim^d T^*(\pi'_d(c_s c_{n+1}), \sigma_b)$

- Substituting back

proving hypothesis

Finishing Proof

- Take $\sigma_a = \sigma_b = \sigma_0$, so from claim proved by induction,

$$T^*(c_s, \sigma_0) \sim^d T^*(\pi'_d(c_s), \sigma_0)$$

- By previous lemma, as *X* (and so $\sim^d$) output consistent, then *X* is noninterference-secure with respect to policy *r*