

# ECS 235B Module 54

## Analyzing Covert Channels

# Analyzing Covert Channels

- Policy and operational issues determine how dangerous it is
  - What follows assumes a policy saying all covert channels are a problem
- *Amount* of information that can be transmitted affects how serious a problem a covert channel is
  - 1 bit per hour: probably harmless in most circumstances
  - 1,000,000 bits per second: probably dangerous in most circumstances
  - Begin here . . .

# Measuring Capacity

- Intuitively, difference between unmodulated, modulated channel
  - Normal uncertainty in channel is 8 bits
  - Attacker modulates channel to send information, reducing uncertainty to 5 bits
  - Covert channel capacity is 3 bits
    - Modulation in effect fixes those bits

# Formally

- Inputs:
  - $A$  input from Alice (sender)
  - $V$  input from everyone else
  - $X$  output of channel
- Capacity measures uncertainty in  $X$  given  $A$
- In other terms: maximize

$$I(A; X) = H(X) - H(X | A)$$

with respect to  $A$

# Noninterference and Covert Channels

- If  $A$ ,  $V$  are independent and  $A$  noninterfering with  $X$ , then  $I(A; X) = 0$
- Why? Intuition is that  $A$  and  $X$  are independent
  - If so, then only  $V$  affects  $X$  (noninterference)
  - So information from  $A$  cannot affect  $X$  unless  $A$  influences  $V$
  - But  $A$  and  $V$  are independent, so information from  $A$  does not affect  $X$
- But noninterference is not necessary

# Example: Noninterference Not Necessary

- System has 1 bit of state; 3 inputs  $I_A, I_B, I_C$ ; one output  $O_X$
- Each input flips state, and state's value is then output
  - System initially in state 0
- $w$  sequence of inputs corresponding to output  $x(w) = \text{length}(w) \bmod 2$ 
  - $I_A$  not noninterfering as deleting its inputs may change output
- Define terms
  - $W$  random variable corresponding to length of input sequences
  - $A$  random variable corresponding to length of input sequences contributed by  $I_A$ ;  $V$  random variable corresponding to other contributions;  $A, V$  independent
  - $X$  random variable corresponding to output state

# Two Cases

- $V = 0$ ; then as  $W = (A + V) \bmod 2$ ,  $W = A$ , and so  $A, W$  not independent; neither are  $A, X$ . So if  $V = 0$ ,  $I(A, X) \neq 0$
- $I_B, I_C$  produce inputs such that  $p(V=0) = p(V=1) = 0.5$ ; then

$$p(X=x) = p(V=x, A=0) + p(V = 1 - x, A = 1)$$

Because  $A, V$  independent, this becomes

$$p(X=x) = p(V=x, A=0) + p(V = 1 - x)p(A = 1)$$

and so  $p(X=x) = 0.5$ . Also,

$$p(X=x \mid A=a) = p(X = (a + x) \bmod 2) = 0.5$$

establishing  $A, X$  independent; so  $I(A, X) = 0$

# Meaning

- Note  $A, X$  noninterfering, and  $I(A; X) = 0$
- So covert channel capacity is 0 if either of the following hold:
  - Input is noninterfering with output; or
  - Input comes from independent sources, all possible values from at least one source are equally probable

# Example (More Formally)

- If  $A, V$  independent, take  $p=p(A=0), q=p(V=0)$ :
  - $p(A=0, V=0) = pq$
  - $p(A=1, V=0) = (1-p)q$
  - $p(A=0, V=1) = p(1-q)$
  - $p(A=1, V=1) = (1-p)(1-q)$
- So
  - $p(X=0) = p(A=0, V=0) + p(A=1, V=1) = pq + (1-p)(1-q)$
  - $p(X=1) = p(A=0, V=1) + p(A=1, V=0) = (1-p)q + p(1-q)$

# Example (*con't*)

- Also:
  - $p(X=0 | A=0) = q$
  - $p(X=0 | A=1) = 1-q$
  - $p(X=1 | A=0) = 1-q$
  - $p(X=1 | A=1) = q$
- So you can compute:
  - $H(X) = -[(1-p)q + p(1-q)] \lg [(1-p)q + p(1-q)]$
  - $H(X|A) = -q \lg q - (1-q) \lg (1-q)$
  - $I(A;X) = H(X) - H(X|A)$

## Example (*con't*)

- So  $I(A; X) = - [pq + (1 - p)(1 - q)] \lg [pq + (1 - p)(1 - q)] - [(1 - p)q + p(1 - q)] \lg [(1 - p)q + p(1 - q)] + q \lg q + (1 - q) \lg (1 - q)$
- Maximum when  $p = 0.5$ ; then
$$I(A; X) = 1 + q \lg q + (1 - q) \lg (1 - q) = 1 - H(V)$$
- So, if  $q = 0$  (meaning  $V$  is constant) then  $I(A; X) = 1$
- Also, if  $q = p = 0.5$ ,  $I(A; X) = 0$

# Analyzing Capacity

- Assume a noisy channel
- Examine covert channel in MLS database that uses replication to ensure availability
  - 2-phase commit protocol ensures atomicity
  - *Coordinator* process manages global execution
  - *Participant* processes do everything else

# How It Works

- Coordinator sends message to each participant asking whether to abort or commit transaction
  - If any says “abort”, coordinator stops
- Coordinator gathers replies
  - If all say “commit”, sends commit messages back to participants
  - If any says “abort”, sends abort messages back to participants
  - Each participant that sent commit waits for reply; on receipt, acts accordingly

# Exceptions

- Protocol times out, causing party to act as if transaction aborted, when:
  - Coordinator doesn't receive reply from participant
  - Participant who sends a commit doesn't receive reply from coordinator

# Covert Channel Here

- Two types of components
  - One at *Low* security level, other at *High*
- Low component begins 2-phase commit
  - Both *High*, *Low* components must cooperate in the 2-phase commit protocol
- *High* sends information to *Low* by selectively aborting transactions
  - Can send abort messages
  - Can just not do anything

# Note

- If transaction *always* succeeded except when *High* component sending information, channel not noisy
  - Capacity would be 1 bit per trial
  - But channel noisy as transactions may abort for reasons *other* than the sending of information

# Analysis

- $X$  random variable: what *High* user wants to send
  - Assume abort is 1, commit is 0
  - $p = p(X=0)$  probability *High* sends 0
- $A$  random variable: what *Low* receives
  - For noiseless channel  $X = A$
- $n+2$  users
  - Sender, receiver,  $n$  others that act independently of one another
  - $q$  probability of transaction aborting at any of these  $n$  users

# Basic Probabilities

- Probabilities of receiving given sending
  - $p(A=0|X=0) = (1-q)^n$
  - $p(A=1|X=0) = 1-(1-q)^n$
  - $p(A=0|X=1) = 0$
  - $p(A=1|X=1) = 1$
- So probabilities of receiving values:
  - $p(A=0) = p(1-q)^n$
  - $p(A=1) = 1-p(1-q)^n$

# More Probabilities

- Given sending, what is receiving?
  - $p(X=0 | A=0) = 1$
  - $p(X=1 | A=0) = 0$
  - $p(X=0 | A=1) = p[1-(1-q)^n] / [1-p(1-q)^n]$
  - $p(X=1 | A=1) = (1-p) / [1-p(1-q)^n]$

# Entropies

You can compute these:

- $H(X) = -p \lg p - (1-p) \lg (1-p)$
- $H(X|A) = -p[1-(1-q)^n] \lg p - p[1-(1-q)^n] \lg [1-(1-q)^n] + [1-p(1-q)^n] \lg [1-p(1-q)^n] - (1-p) \lg (1-p)$
- $I(A;X) = -p(1-q)^n \lg p + p[1-(1-q)^n] \lg [1-(1-q)^n] - [1-p(1-q)^n] \lg [1-p(1-q)^n]$

# Capacity

- Maximize this with respect to  $p$  (probability that *High* sends 0)
  - Notation:  $m = (1-q)^n$ ,  $M = (1-m)^{(1-m)}$
  - Maximum when  $p = M^{(1/m)} / (M^{(1/m)}m+1)$

- Capacity is:

$$I(A;X) = \frac{Mm \lg p + M(1-m) \lg (1-m) + \lg (Mm+1)}{(Mm+1)}$$