Questions

1. (30 points) Let $L = (S_L, \leq_L)$ be a lattice. Define:
   
   (a) $S_{IL} = \{ [a, b] \mid a, b \in S_L \land a \leq_L b \}$
   
   (b) $\leq_{IL} = \{ ([a_1, b_1], [a_2, b_2]) \mid a_1 \leq_L a_2 \land b_1 \leq_L b_2 \}$
   
   (c) $\text{lub}_{IL}([a_1, b_1], [a_2, b_2]) = (\text{lub}_L(a_1, a_2), \text{lub}_L(b_1, b_2))$
   
   (d) $\text{glb}_{IL}([a_1, b_1], [a_2, b_2]) = (\text{glb}_L(a_1, a_2), \text{glb}_L(b_1, b_2))$

   Prove that the structure $IL = (S_{IL}, \leq_{IL})$ is a lattice.

2. (30 points) The following system call adds read permission for a process $(for\_pid)$ if the caller $(call\_pid)$ owns the file, and does nothing otherwise. (The operating system supplies $call\_pid$; the caller supplies the two latter parameters.)

   ```
   function addread (call\_pid, for\_pid: process\_id; fid: file\_id): integer;
   begin
      if (call\_pid = filelist[fid].owner) then
         addright (filelist[fid].access\_control\_list, for\_pid, "r")
      result := (call\_pid = filelist[fid].owner);
      return result
   end.
   ```

   (a) Is the variable $result$ directly or indirectly visible, or not visible?

   (b) Is the variable $filelist[fid].owner$ directly or indirectly visible, or not visible?

   (c) Is the variable $filelist[fid].access\_control$ directly or indirectly visible, or not visible?

3. (40 points) Section 18.3.2.3 derives a formula for $I(A;X)$. Prove that this formula is a maximum with respect to $p$ when $p = \frac{M^m}{1+Mm}$, with $M$ and $m$ as defined in that section. (The value of $p$ in the book is incorrect.)