ECS 235B Module 11
Expressiveness
Expressive Power

• How do the sets of systems that models can describe compare?
  • If HRU equivalent to SPM, SPM provides more specific answer to safety question
  • If HRU describes more systems, SPM applies only to the systems it can describe
HRU vs. SPM

• SPM more abstract
  • Analyses focus on limits of model, not details of representation
• HRU allows revocation
  • SMP has no equivalent to delete, destroy
• HRU allows multiparent creates
  • SMP cannot express multiparent creates easily, and not at all if the parents are of different types because `can•create` allows for only one type of creator
Multiparent Create

• Solves mutual suspicion problem
  • Create proxy jointly, each gives it needed rights

• In HRU:

  command multicreate(s₀, s₁, o)
  if r in a[s₀, s₁] and r in a[s₁, s₀]
  then
     create object o;
     enter r into a[s₀, o];
     enter r into a[s₁, o];
  end
SPM and Multiparent Create

• \textit{cc} extended in obvious way
  • \textit{cc} $\subseteq TS \times \ldots \times TS \times T$

• Symbols
  • $X_1, \ldots, X_n$ parents, $Y$ created
  • $R_{1,i}, R_{2,i}, R_3, R_{4,i} \subseteq R$

• Rules
  • $cr_{p,i}(\tau(X_1), \ldots, \tau(X_n)) = Y/R_{1,1} \cup X_i/R_{2,i}$
  • $cr_{C}(\tau(X_1), \ldots, \tau(X_n)) = Y/R_3 \cup X_1/R_{4,1} \cup \ldots \cup X_n/R_{4,n}$
Example

• Anna, Bill must do something cooperatively
  • But they don’t trust each other

• Jointly create a proxy
  • Each gives proxy only necessary rights

• In ESPM:
  • Anna, Bill type $a$; proxy type $p$; right $x \in R$
  • $cc(a, a) = p$
  • $cr_{Anna}(a, a, p) = cr_{Bill}(a, a, p) = \emptyset$
  • $cr_{proxy}(a, a, p) = \{\text{Anna}/x, \text{Bill}/x\}$
2-Parent Joint Create Suffices

• Goal: emulate 3-parent joint create with 2-parent joint create

• Definition of 3-parent joint create (subjects $P_1$, $P_2$, $P_3$; child $C$):
  • $cc(\tau(P_1), \tau(P_2), \tau(P_3)) = Z \subseteq T$
  • $cr_{P_1}(\tau(P_1), \tau(P_2), \tau(P_3)) = C/R_{1,1} \cup P_1/R_{2,1}$
  • $cr_{P_2}(\tau(P_1), \tau(P_2), \tau(P_3)) = C/R_{2,1} \cup P_2/R_{2,2}$
  • $cr_{P_3}(\tau(P_1), \tau(P_2), \tau(P_3)) = C/R_{3,1} \cup P_3/R_{2,3}$
General Approach

• Define agents for parents and child
  • Agents act as surrogates for parents
  • If create fails, parents have no extra rights
  • If create succeeds, parents, child have exactly same rights as in 3-parent creates
    • Only extra rights are to agents (which are never used again, and so these rights are irrelevant)
Entities and Types

• Parents $P_1, P_2, P_3$ have types $p_1, p_2, p_3$
• Child $C$ of type $c$
• Parent agents $A_1, A_2, A_3$ of types $a_1, a_2, a_3$
• Child agent $S$ of type $s$
• Type $t$ is parentage
  • if $X/t \in \text{dom}(Y)$, $X$ is $Y$’s parent
• Types $t, a_1, a_2, a_3, s$ are new types
can\textbullet create

- Following added to can\textbullet create:
  - $cc(p_1) = a_1$
  - $cc(p_2, a_1) = a_2$
  - $cc(p_3, a_2) = a_3$
    - Parents creating their agents; note agents have maximum of 2 parents
  - $cc(a_3) = s$
    - Agent of all parents creates agent of child
  - $cc(s) = c$
    - Agent of child creates child
Creation Rules

• Following added to create rule:
  • $cr_p(p_1, a_1) = \emptyset$
  • $cr_c(p_1, a_1) = p_1/Rtc$
    • Agent’s parent set to creating parent; agent has all rights over parent
  • $cr_{p_{first}}(p_2, a_1, a_2) = \emptyset$
  • $cr_{p_{second}}(p_2, a_1, a_2) = \emptyset$
  • $cr_c(p_2, a_1, a_2) = p_2/Rtc \cup a_1/tc$
    • Agent’s parent set to creating parent and agent; agent has all rights over parent (but not over agent)
Creation Rules

- \( cr_{p_{\text{first}}}(p_3, a_2, a_3) = \emptyset \)
- \( cr_{p_{\text{second}}}(p_3, a_2, a_3) = \emptyset \)
- \( cr_{c}(p_3, a_2, a_3) = \frac{p_3}{Rtc} \cup \frac{a_2}{tc} \)
  - Agent’s parent set to creating parent and agent; agent has all rights over parent (but not over agent)
- \( cr_{p}(a_3, s) = \emptyset \)
- \( cr_{c}(a_3, s) = \frac{a_3}{tc} \)
  - Child’s agent has third agent as parent \( cr_{p}(a_3, s) = \emptyset \)
- \( cr_{p}(s, c) = \frac{C}{Rtc} \)
- \( cr_{c}(s, c) = \frac{c}{R_3 t} \)
  - Child’s agent gets full rights over child; child gets \( R_3 \) rights over agent
Link Predicates

• Idea: no tickets to parents until child created
  • Done by requiring each agent to have its own parent rights
  • \( \text{link}_1(A_2, A_1) = A_1/t \in \text{dom}(A_2) \land A_2/t \in \text{dom}(A_2) \)
  • \( \text{link}_1(A_3, A_2) = A_2/t \in \text{dom}(A_3) \land A_3/t \in \text{dom}(A_3) \)
  • \( \text{link}_2(S, A_3) = A_3/t \in \text{dom}(S) \land C/t \in \text{dom}(C) \)
  • \( \text{link}_3(A_1, C) = C/t \in \text{dom}(A_1) \)
  • \( \text{link}_3(A_2, C) = C/t \in \text{dom}(A_2) \)
  • \( \text{link}_3(A_3, C) = C/t \in \text{dom}(A_3) \)
  • \( \text{link}_4(A_1, P_1) = P_1/t \in \text{dom}(A_1) \land A_1/t \in \text{dom}(A_1) \)
  • \( \text{link}_4(A_2, P_2) = P_2/t \in \text{dom}(A_2) \land A_2/t \in \text{dom}(A_2) \)
  • \( \text{link}_4(A_3, P_3) = P_3/t \in \text{dom}(A_3) \land A_3/t \in \text{dom}(A_3) \)
Filter Functions

• $f_1(a_2, a_1) = a_1/t \cup c/Rtc$
• $f_1(a_3, a_2) = a_2/t \cup c/Rtc$
• $f_2(s, a_3) = a_3/t \cup c/Rtc$
• $f_3(a_1, c) = p_1/R_{4,1}$
• $f_3(a_2, c) = p_2/R_{4,2}$
• $f_3(a_3, c) = p_3/R_{4,3}$
• $f_4(a_1, p_1) = c/R_{1,1} \cup p_1/R_{2,1}$
• $f_4(a_2, p_2) = c/R_{1,2} \cup p_2/R_{2,2}$
• $f_4(a_3, p_3) = c/R_{1,3} \cup p_3/R_{2,3}$
Construction

Create $A_1$, $A_2$, $A_3$, $S$, $C$; then

- $P_1$ has no relevant tickets
- $P_2$ has no relevant tickets
- $P_3$ has no relevant tickets
- $A_1$ has $P_1/Rtc$
- $A_2$ has $P_2/Rtc \cup A_1/tc$
- $A_3$ has $P_3/Rtc \cup A_2/tc$
- $S$ has $A_3/tc \cup C/Rtc$
- $C$ has $C/R_3t$
Construction

- Only $\text{link}_2(S, A_3)$ true $\Rightarrow$ apply $f_2$
  - $A_3$ has $P_3/Rtc \cup A_2/t \cup A_3/t \cup C/Rtc$
- Now $\text{link}_1(A_3, A_2)$ true $\Rightarrow$ apply $f_1$
  - $A_2$ has $P_2/Rtc \cup A_1/tc \cup A_2/t \cup C/Rtc$
- Now $\text{link}_1(A_2, A_1)$ true $\Rightarrow$ apply $f_1$
  - $A_1$ has $P_2/Rtc \cup A_1/t \cup C/Rtc$
- Now all $\text{link}_3$s true $\Rightarrow$ apply $f_3$
  - $C$ has $C/R_3 \cup P_1/R_{4,1} \cup P_2/R_{4,2} \cup P_3/R_{4,3}$
Finish Construction

• Now \( \text{link}_4 \) is true \( \Rightarrow \) apply \( f_4 \)
  - \( P_1 \) has \( C/R_{1,1} \cup P_1/R_{2,1} \)
  - \( P_2 \) has \( C/R_{1,2} \cup P_2/R_{2,2} \)
  - \( P_3 \) has \( C/R_{1,3} \cup P_3/R_{2,3} \)

• 3-parent joint create gives same rights to \( P_1, P_2, P_3, C \)
• If create of \( C \) fails, \( \text{link}_2 \) fails, so construction fails
Theorem

• The two-parent joint creation operation can implement an $n$-parent joint creation operation with a fixed number of additional types and rights, and augmentations to the link predicates and filter functions.

• **Proof**: by construction, as above
  • Difference is that the two systems need not start at the same initial state
Theorems

• Monotonic ESPM and the monotonic HRU model are equivalent.

• Safety question in ESPM also decidable if acyclic attenuating scheme
  • Proof similar to that for SPM
Expressiveness

• Graph-based representation to compare models

• Graph
  • Vertex: represents entity, has static type
  • Edge: represents right, has static type

• Graph rewriting rules:
  • Initial state operations create graph in a particular state
  • Node creation operations add nodes, incoming edges
  • Edge adding operations add new edges between existing vertices
Example: 3-Parent Joint Creation

• Simulate with 2-parent
  • Nodes $P_1, P_2, P_3$ parents
  • Create node $C$ with type $c$ with edges of type $e$
  • Add node $A_1$ of type $a$ and edge from $P_1$ to $A_1$ of type $e'$
Next Step

- $A_1, P_2$ create $A_2$; $A_2, P_3$ create $A_3$
- Type of nodes, edges are $a$ and $e'$
Next Step

• $A_3$ creates $S$, of type $a$
• $S$ creates $C$, of type $c$
Last Step

• Edge adding operations:
  • $P_1 \rightarrow A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow S \rightarrow C$: $P_1$ to $C$ edge type $e$
  • $P_2 \rightarrow A_2 \rightarrow A_3 \rightarrow S \rightarrow C$: $P_2$ to $C$ edge type $e$
  • $P_3 \rightarrow A_3 \rightarrow S \rightarrow C$: $P_3$ to $C$ edge type $e$
Definitions

- **Scheme**: graph representation as above
- **Model**: set of schemes
- Schemes $A, B$ correspond if graph for both is identical when all nodes with types not in $A$ and edges with types in $A$ are deleted
Example

• Above 2-parent joint creation simulation in scheme \textit{TWO}

• Equivalent to 3-parent joint creation scheme \textit{THREE} in which $P_1, P_2, P_3, C$ are of same type as in \textit{TWO}, and edges from $P_1, P_2, P_3$ to $C$ are of type $e$, and no types $a$ and $e'$ exist in \textit{TWO}
Simulation

Scheme $A$ simulates scheme $B$ iff

• every state $B$ can reach has a corresponding state in $A$ that $A$ can reach; and

• every state that $A$ can reach either corresponds to a state $B$ can reach, or has a successor state that corresponds to a state $B$ can reach
  • The last means that $A$ can have intermediate states not corresponding to states in $B$, like the intermediate ones in $TWO$ in the simulation of $THREE$
Expressive Power

• If there is a scheme in $MA$ that no scheme in $MB$ can simulate, $MB$ less expressive than $MA$

• If every scheme in $MA$ can be simulated by a scheme in $MB$, $MB$ as expressive as $MA$

• If $MA$ as expressive as $MB$ and vice versa, $MA$ and $MB$ equivalent
Example

• Scheme $A$ in model $M$
  • Nodes $X_1, X_2, X_3$
  • 2-parent joint create
  • 1 node type, 1 edge type
  • No edge adding operations
  • Initial state: $X_1, X_2, X_3$, no edges

• Scheme $B$ in model $N$
  • All same as $A$ except no 2-parent joint create
  • 1-parent create

• Which is more expressive?
Can A Simulate $B$?

• Scheme A simulates 1-parent create: have both parents be same node
  • Model $M$ as expressive as model $N$
Can $B$ Simulate $A$?

• Suppose $X_1, X_2$ jointly create $Y$ in $A$
  • Edges from $X_1, X_2$ to $Y$, no edge from $X_3$ to $Y$
• Can $B$ simulate this?
  • Without loss of generality, $X_1$ creates $Y$
  • Must have edge adding operation to add edge from $X_2$ to $Y$
  • One type of node, one type of edge, so operation can add edge between any 2 nodes
No

• All nodes in $A$ have even number of incoming edges
  • 2-parent create adds 2 incoming edges
• Edge adding operation in $B$ that can edge from $X_2$ to $C$ can add one from $X_3$ to $C$
  • $A$ cannot enter this state
  • $B$ cannot transition to a state in which $Y$ has even number of incoming edges
    • No remove rule
• So $B$ cannot simulate $A$; $N$ less expressive than $M$
Theorem

• Monotonic single-parent models are less expressive than monotonic multiparent models

• Proof by contradiction
  • Scheme $A$ is multiparent model
  • Scheme $B$ is single parent create
  • Claim: $B$ can simulate $A$, without assumption that they start in the same initial state
    • Note: example assumed same initial state
Outline of Proof

• $X_1, X_2$ nodes in $A$
  • They create $Y_1, Y_2, Y_3$ using multiparent create rule
  • $Y_1, Y_2$ create $Z$, again using multiparent create rule
  • Note: no edge from $Y_3$ to $Z$ can be added, as $A$ has no edge-adding operation
Outline of Proof

- \( W, X_1, X_2 \) nodes in \( B \)
  - \( W \) creates \( Y_1, Y_2, Y_3 \) using single parent create rule, and adds edges for \( X_1, X_2 \) to all using edge adding rule
  - \( Y_1 \) creates \( Z \), again using single parent create rule; now must add edge from \( Y_2 \) to \( Z \) to simulate \( A \)
  - Use same edge adding rule to add edge from \( Y_3 \) to \( Z \): cannot duplicate this in scheme \( A \)!
Meaning

• Scheme $B$ cannot simulate scheme $A$, contradicting hypothesis

• ESPM more expressive than SPM
  • ESPM multiparent and monotonic
  • SPM monotonic but single parent