

# ECS 235B Module 48

## Introduction to Information Flow

# Basics

- Bell-LaPadula Model embodies information flow policy
  - Given compartments  $A, B$ , info can flow from  $A$  to  $B$  iff  $B \text{ dom } A$
- So does Biba Model
  - Given compartments  $A, B$ , info can flow from  $A$  to  $B$  iff  $A \text{ dom } B$
- Variables  $x, y$  assigned compartments  $\underline{x}, \underline{y}$  as well as values
  - Confidentiality (Bell-LaPadula): if  $\underline{x} = A, \underline{y} = B$ , and  $B \text{ dom } A$ , then  $y := x$  allowed but not  $x := y$
  - Integrity (Biba): if  $\underline{x} = A, \underline{y} = B$ , and  $A \text{ dom } B$ , then  $x := y$  allowed but not  $y := x$
- For now, focus on confidentiality (Bell-LaPadula)
  - We'll get to integrity later

# Entropy and Information Flow

- Idea: information flows from  $x$  to  $y$  as a result of a sequence of commands  $c$  if you can deduce information about  $x$  before  $c$  from the value in  $y$  after  $c$
- Formally:
  - $s$  time before execution of  $c$ ,  $t$  time after
  - $H(x_s | y_t) < H(x_s | y_s)$
  - If no  $y$  at time  $s$ , then  $H(x_s | y_t) < H(x_s)$

# Example 1

- Command is  $x := y + z$ ; where:
  - $x$  does not exist initially (that is, has no value)
  - $0 \leq y \leq 7$ , equal probability
  - $z = 1$  with probability  $1/2$ ,  $z = 2$  or  $3$  with probability  $1/4$  each
- $s$  state before command executed;  $t$ , after; so
  - $H(y_s) = H(y_t) = -8(1/8) \lg(1/8) = 3$
- You can show that  $H(y_s | x_t) = (3/32) \lg 3 + 9/8 \approx 1.274 < 3 = H(y_s)$ 
  - Thus, information flows from  $y$  to  $x$

# Example 2

- Command is

**if  $x = 1$  then  $y := 0$  else  $y := 1$ ;**

where  $x, y$  equally likely to be either 0 or 1

- $H(x_s) = 1$  as  $x$  can be either 0 or 1 with equal probability
- $H(x_s | y_t) = 0$  as if  $y_t = 1$  then  $x_s = 0$  and vice versa
  - Thus,  $H(x_s | y_t) = 0 < 1 = H(x_s)$
- So information flowed from  $x$  to  $y$

# Implicit Flow of Information

- Information flows from  $x$  to  $y$  without an *explicit* assignment of the form  $y := f(x)$ 
  - $f(x)$  an arithmetic expression with variable  $x$
- Example from previous slide:  
**if  $x = 1$  then  $y := 0$  else  $y := 1$ ;**
- So must look for implicit flows of information to analyze program

# Notation

- $\underline{x}$  means class of  $x$ 
  - In Bell-LaPadula based system, same as “label of security compartment to which  $x$  belongs”
- $\underline{x} \leq \underline{y}$  means “information can flow from an element in class of  $x$  to an element in class of  $y$ ”
  - Or, “information with a label placing it in class  $\underline{x}$  can flow into class  $\underline{y}$ ”