Lecture 18 Outline (May 8, 2015)

1. Greetings and felicitations!

2. Classical Cryptography
   a. Monoalphabetic (simple substitution): \( f(a) = a + k \mod n \)
   b. Example: Caesar with \( k = 3 \), RENAISSANCE \( \rightarrow \) UHQLDLVVDQFH
   c. Polyalphabetic: Vigenère, \( f_i(a) = a + k_i \mod n \)
   d. Cryptanalysis: first do index of coincidence to see if it is monoalphabetic or polyalphabetic, then Kasiski method.
   e. Problem: eliminate periodicity of key

3. Long key generation
   a. Autokey cipher:
      \[ M = \text{THE Treasure is Buried} \]
      \[ K = \text{HELLO THE Treasure is Buried} \]
      \[ C = \text{ALPEFXHWNIIKVLVQWE} \]
   b. Running-key cipher:
      \[ M = \text{THE Treasure is Buried} \]
      \[ K = \text{THE SECOND CIPHER IS AN} \]
      \[ C = \text{MOILVGOFXTMXZFLZAEQ} \]
   c. Perfect secrecy: when the probability of computing the plaintext message is the same whether or not you have the ciphertext
   d. Only cipher with perfect secrecy: one-time pads; \( C = \text{AZPR} \); is that \textit{DOIT} or \textit{DONT}?

4. Product ciphers: DES, AES

5. Public-Key Cryptography
   a. Basic idea: 2 keys, one private, one public
   b. Cryptosystem must satisfy:
      i. Given public key, computationally infeasible to get private key;
      ii. Cipher withstands chosen plaintext attack;
      iii. Encryption, decryption computationally feasible (\textit{note}: commutativity not required)
   c. Benefits: can give confidentiality or authentication or both

6. Use of public key cryptosystem
   a. Normally used as key interchange system to exchange secret keys (cheap)
   b. Then use secret key system (too expensive to use public key cryptosystem for this)

7. RSA
   a. Provides both authenticity and confidentiality
   b. Go through algorithm:
      Idea: \( C = M^e \mod n, M = C^d \mod n \), with \( e d \mod \phi(n) = 1 \)
      Public key is \((e, n)\); private key is \(d\). Choose \( n = pq \); then \( \phi(n) = (p - 1)(q - 1) \).
   c. Example: \( p = 5, q = 7 \); then \( n = 35, \phi(n) = (5 - 1)(7 - 1) = 24 \). Pick \( d = 11 \). Then \( ed \mod \phi(n) = 1 \), so \( e = 11 \)
      To encipher 2, \( C = M^e \mod n = 2^{11} \mod 35 = 2048 \mod 35 = 18 \), and \( M = C^d \mod n = 18^{11} \mod 35 = 2 \).
   d. Example: \( p = 53, q = 61 \); then \( n = 3233, \phi(n) = (53 - 1)(61 - 1) = 3120 \). Pick \( d = 791 \). Then \( e = 71 \)
      To encipher \( M = \text{RENAISSANCE} \), use the mapping \( A = 00, B = 01, \ldots, Z = 25, \emptyset = 26 \).
      Then: \( M = \text{RE NA IS SA NC EB} = 1704 1300 0818 1800 1302 0426 \)
      So: \( C = (1704)^{71} \mod 3233 = 3106; \ldots = 3106 0100 0931 2691 1984 2927 \)