Lecture 10, April 20, 2026

Overview of the DES

- A block cipher:
  - encrypts blocks of 64 bits using a 64 bit key
  - outputs 64 bits of ciphertext
- A product cipher
  - basic unit is the bit
  - performs both substitution and transposition (permutation) on the bits
- Cipher consists of 16 rounds (iterations) each with a 48 bit round key generated from the user-supplied key

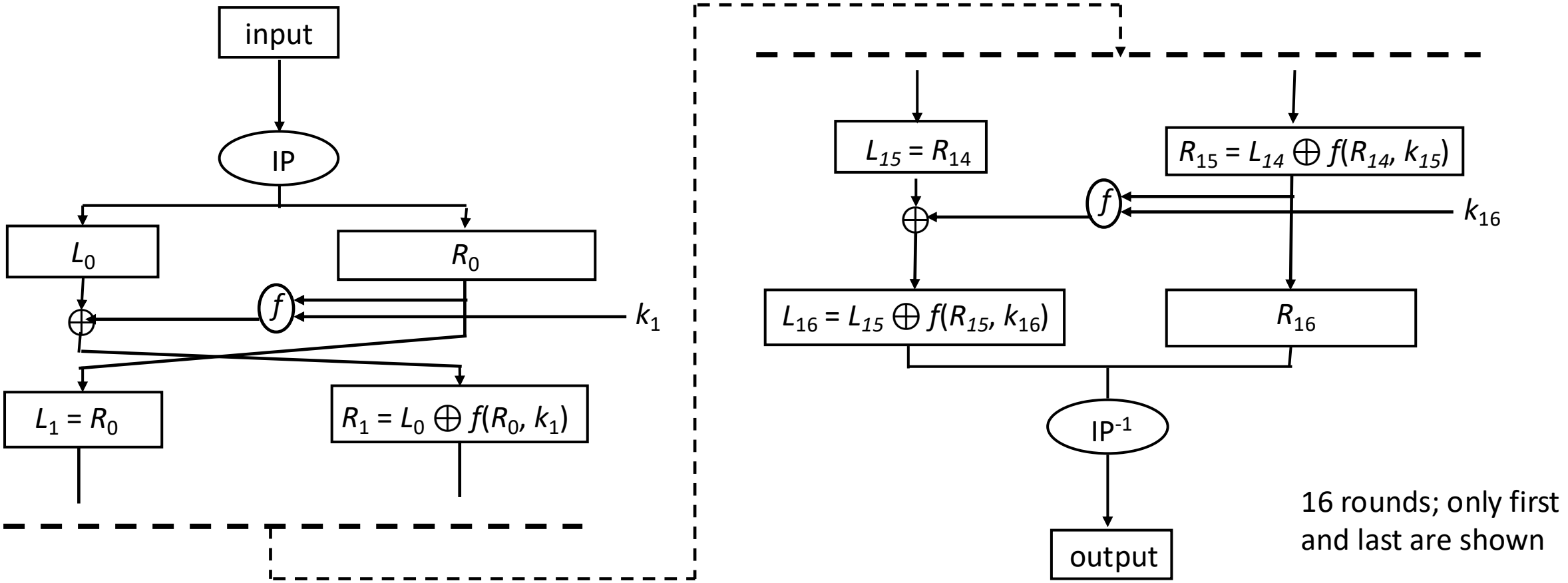
Structure of the DES

- Input is first permuted, then split into left half (L) and right half (R), each 32 bits
- Round begins; R and round key run through function *f*, then xor'ed with L
  - *f* expands R to 48 bits, xors with round key, and then each 6 bits of this are run through S-boxes (substitution boxes), each of which gives 4 bits of output
  - Those 32 bits are permuted and this is the output of *f*
- R and L swapped, ending the round
  - Swapping does not occur in the last round
- After last round, L and R combined, permuted, forming DES output

Main Algorithm

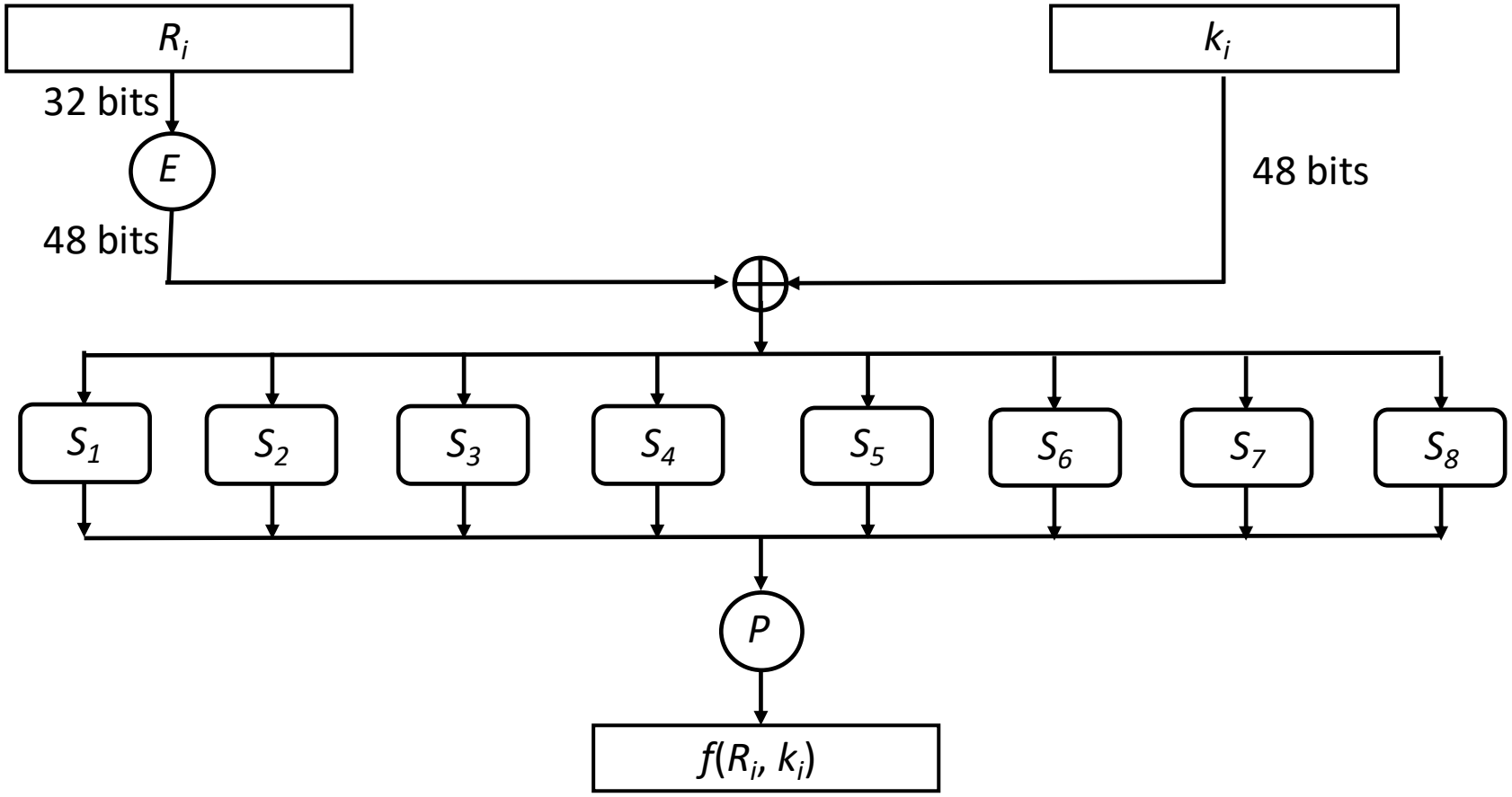
- Key permuted, split into 2 28-bit parts
  - Each part rotated left by 1 or 2 bits
  - Then the halves combined, permuted, and 48 bits output (*round key*)
- Input permuted, split into 2 32-bit parts
  - Right half, round key fed into function *f*
  - Result of this xor'ed with left half
  - This left half becomes right half, right half becomes left half, as input to next round (but in the last round, this does not occur)
- After 16 rounds, halves combined, then permuted and that is output
  - Permutation here is inverse of initial input permutation

DES Algorithm: Rounds

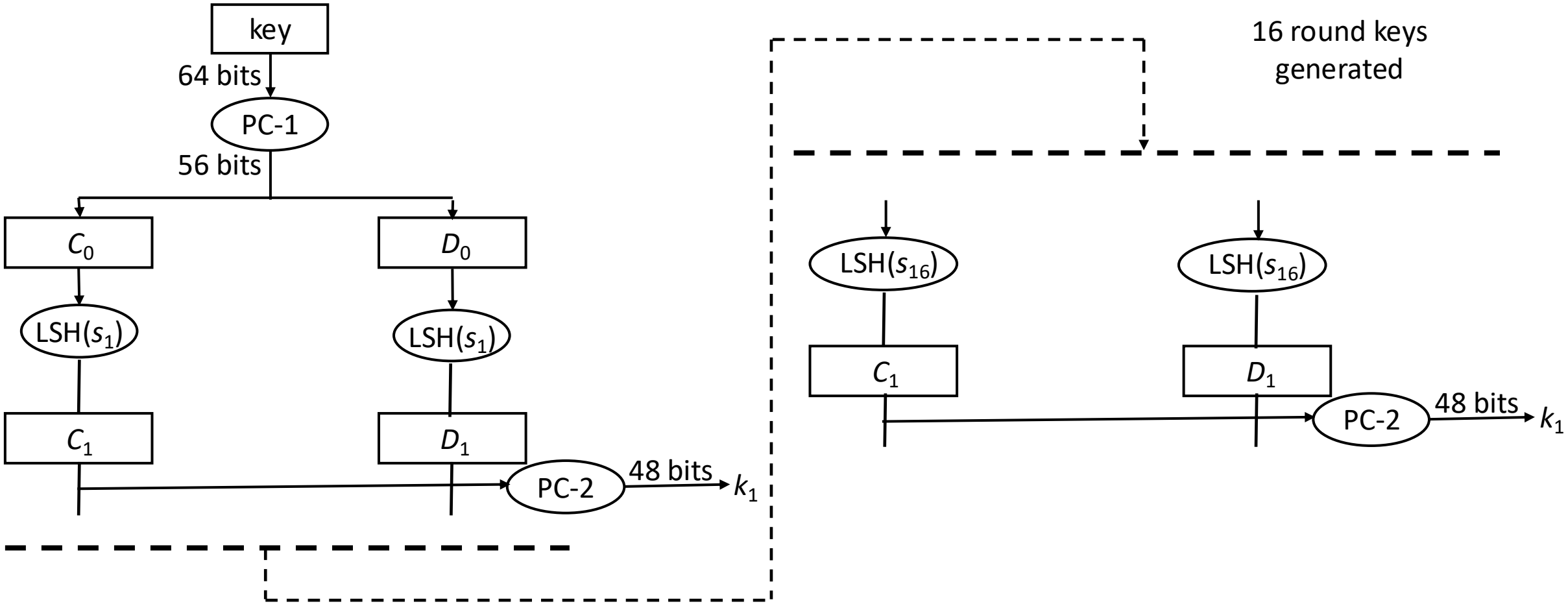


16 rounds; only first and last are shown

DES Algorithm: *f*



DES Algorithm: Round Key Generation



Controversy

- Considered too weak
  - Diffie, Hellman said in a few years technology would allow DES to be broken in days
    - Design using 1999 technology published
- Design decisions not public
  - S-boxes may have backdoors

Undesirable Properties

- 4 weak keys
  - They are their own inverses
- 12 semi-weak keys
  - Each has another semi-weak key as inverse
- Complementation property
  - $\text{DES}_k(m) = c \Rightarrow \text{DES}_{k'}(m') = c'$
- S-boxes exhibit irregular properties
  - Distribution of odd, even numbers non-random
  - Outputs of fourth box depends on input to third box

Differential Cryptanalysis

- A chosen ciphertext attack
  - Requires 2^{47} plaintext, ciphertext pairs
- Revealed several properties
  - Small changes in S-boxes reduced the number of pairs needed
  - Making every bit of the round keys independent did not impede attack
- Linear cryptanalysis improves result
  - Requires 2^{43} plaintext, ciphertext pairs

DES Modes

- Electronic Code Book Mode (ECB)
  - Encipher each block independently
- Cipher Block Chaining Mode (CBC)
  - Xor each block with previous ciphertext block
  - Requires an initialization vector for the first one
- Encrypt-Decrypt-Encrypt (2 keys: k, k')
  - $c = \mathrm{DES}_k(\mathrm{DES}_{k'}^{-1}(\mathrm{DES}_k(m)))$
- Triple DES(3 keys: k, k', k'')
  - $c = \mathrm{DES}_k(\mathrm{DES}_{k'}(\mathrm{DES}_{k''}(m)))$

Current Status of DES

- Design for computer system, associated software that could break any DES-enciphered message in a few days published in 1998
- Several challenges to break DES messages solved using distributed computing
- NIST selected Rijndael as Advanced Encryption Standard, successor to DES
  - Designed to withstand attacks that were successful on DES
- DES officially withdrawn in 2005

Advanced Encryption Standard

- Competition announces in 1997 to select successor to DES
  - Successor needed to be available for use without payment (no royalties, etc.)
  - Successor must encipher 128-bit blocks with keys of lengths 128, 192, and 256
- 3 workshops in which proposed successors were presented, analyzed
- Rijndael selected as successor to DES, called the Advanced Encryption Standard (AES)
  - Other finalists were Twofish, Serpent, RC6, MARS

Overview of the AES

- A block cipher:
  - encrypts blocks of 128 bits using a 128, 192, or 256 bit key
  - outputs 128 bits of ciphertext
- A product cipher
  - basic unit is the bit
  - performs both substitution and transposition (permutation) on the bits
- Cipher consists of rounds (iterations) each with a round key generated from the user-supplied key
  - If 128 bit key, then 10 rounds
  - If 192 bit key, then 12 rounds
  - If 256 bit key, then 14 rounds

Structure of the AES: Encryption

- Input placed into a state array, which is then combined with zeroth round key
  - Treat state array as a 4×4 matrix, each entry being a byte
- Round begins; new values substituted for each byte of the state array
- Rows then cyclically shifted
- Each column independently altered
  - Not done in last round
- Result xor'ed with round key
- After last round, state array is the encrypted input

Structure of the AES: Decryption

- Round key schedule reversed
- Input placed into a state array, which is then combined with zeroth round key (of reversed schedule)
- Round begins; rows cyclically shifted, then new values substituted for each byte of the state array
  - Inverse rotation, substitution of encryption
- Result xor'ed with round key (of reversed schedule)
- Each column independently altered
  - Inverse of encryption; this is not done in last round
- After last round, state array is the decrypted input

Analysis of AES

- Designed to withstand attacks that the DES is vulnerable to
- All details of design made public, unlike with the DES
  - In particular, those of the substitutions (S-boxes) were described
- After 2 successive rounds, every bit in the state array depends an every bit in the state array 2 rounds ago
- No weak, semi-weak keys

AES Modes

- DES modes also work with AES
- EDE and “Triple-AES” not used
  - Extended block size makes this unnecessary
- New counter mode CTR added

Public Key Cryptography

- Two keys
  - *Private key* known only to individual
  - *Public key* available to anyone
    - Public key, private key inverses
- Idea
  - Confidentiality: encipher using public key, decipher using private key
  - Integrity/authentication: encipher using private key, decipher using public one

Requirements

1. It must be computationally easy to encipher or decipher a message given the appropriate key
2. It must be computationally infeasible to derive the private key from the public key
3. It must be computationally infeasible to determine the private key from a chosen plaintext attack

RSA

- First described publicly in 1978
  - Unknown at the time: Clifford Cocks developed a similar cryptosystem in 1973, but it was classified until recently
- Exponentiation cipher
- Relies on the difficulty of determining the number of numbers relatively prime to a large integer *n*

Background

- Totient function ϕ(n)
  - Number of positive integers less than *n* and relatively prime to *n*
  - *Relatively prime* means with no factors in common with *n*
- Example: $\phi(10) = 4$
  - 1, 3, 7, 9 are relatively prime to 10
- Example: $\phi(21) = 12$
  - 1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20 are relatively prime to 21

Algorithm

- Choose two large prime numbers p, q
  - Let $n = pq$; then $\phi(n) = (p–1)(q–1)$
  - Choose $e < n$ such that e is relatively prime to $\phi(n)$.
  - Compute d such that $ed \bmod \phi(n) = 1$
- Public key: (e, n); private key: d
- Encipher: $c = m^e \bmod n$
- Decipher: $m = c^d \bmod n$

Example: Confidentiality

- Take $p = 181$, $q = 1451$, so $n = 262631$ and $\phi(n) = 261000$
- Alice chooses $e = 154993$, making $d = 95857$
- Bob wants to send Alice secret message PUPPIESARESMALL (152015 150804 180017 041812 001111); encipher using public key
  - $152015^{154993} \bmod 262631 = 220160$
  - $150804^{154993} \bmod 262631 = 135824$
  - $180017^{154993} \bmod 262631 = 252355$
  - $041812^{154993} \bmod 262631 = 245799$
  - $001111_{154993} \bmod 262631 = 070707$
- Bob sends 220160 135824 252355 245799 070707
- Alice uses her private key to decipher it