

Lecture 19, May 13, 2026

ECS 235B, Foundations of Computer and Information Security
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Review of Information Flow Notation

- Bell-LaPadula Model embodies information flow policy
 - Given compartments A, B , info can flow from A to B iff $B \text{ dom } A$
- So does Biba Model
 - Given compartments A, B , info can flow from A to B iff $A \text{ dom } B$
- Variables x, y assigned compartments $\underline{x}, \underline{y}$ as well as values
 - Confidentiality (Bell-LaPadula): if $\underline{x} = A, \underline{y} = B$, and $B \text{ dom } A$, then $y := x$ allowed but not $x := y$
 - Integrity (Biba): if $\underline{x} = A, \underline{y} = B$, and $A \text{ dom } B$, then $x := y$ allowed but not $y := x$
- What follows focuses on confidentiality (Bell-LaPadula)

Entropy and Information Flow

- Idea: information flows from x to y as a result of a sequence of commands c if you can deduce information about x before c from the value in y after c
- Formally:
 - s time before execution of c , t time after
 - $H(x_s | y_t) < H(x_s | y_s)$
 - If no y at time s , then $H(x_s | y_t) < H(x_s)$

Example 1

- Command is $x := y + z$; where:
 - x does not exist initially (that is, has no value)
 - $0 \leq y \leq 7$, equal probability
 - $z = 1$ with probability $1/2$, $z = 2$ or 3 with probability $1/4$ each
- s state before command executed; t , after; so
 - $H(y_s) = H(y_t) = -8(1/8) \lg(1/8) = 3$
- You can show that $H(y_s | x_t) = (3/32) \lg 3 + 9/8 \approx 1.274 < 3 = H(y_s)$
 - Thus, information flows from y to x

Example 2

- Command is

if $x = 1$ then $y := 0$ else $y := 1$;

where x, y equally likely to be either 0 or 1

- x can be either 0 or 1 with equal probability, so $H(x_s) = 1$
- If $y_t = 1$ then $x_s = 0$ (and vice versa), so $H(x_s | y_t) = 0$
 - Thus, $H(x_s | y_t) = 0 < 1 = H(x_s)$
- So information flowed from x to y

Implicit Flow of Information

- Information flows from x to y without an *explicit* assignment of the form $y := f(x)$
 - $f(x)$ an arithmetic expression with variable x
- Example from previous slide:
if $x = 1$ then $y := 0$ else $y := 1$;
- So must look for implicit flows of information to analyze program

Notation

- \underline{x} means class of x
 - In Bell-LaPadula based system, same as “label of security compartment to which x belongs”
- $\underline{x} \leq \underline{y}$ means “information can flow from an element in class of x to an element in class of y ”
 - Or, “information with a label placing it in class \underline{x} can flow into class \underline{y} ”

Information Flow Policies

Information flow policies are usually:

- reflexive
 - So information can flow freely among members of a single class
- transitive
 - So if information can flow from class 1 to class 2, and from class 2 to class 3, then information can flow from class 1 to class 3

Non-Transitive Policies

- Betty is a confidant of Anne
- Cathy is a confidant of Betty
 - With transitivity, information flows from Anne to Betty to Cathy
- Anne confides to Betty she is having an affair with Cathy's spouse
 - Transitivity undesirable in this case, probably

Non-Lattice Transitive Policies

- 2 faculty members co-PIs on a grant
 - Equal authority; neither can overrule the other
- Grad students report to faculty members
- Undergrads report to grad students
- Information flow relation is:
 - Reflexive and transitive
- But some elements (people) have no “least upper bound” element
 - What is it for the faculty members?

Confidentiality Policy Model

- Lattice model fails in previous 2 cases
- Generalize: policy $I = (SC_I, \leq_I, join_I)$:
 - SC_I set of security classes
 - \leq_I ordering relation on elements of SC_I
 - $join_I$ function to combine two elements of SC_I
- Example: Bell-LaPadula Model
 - SC_I set of security compartments
 - \leq_I ordering relation *dom*
 - $join_I$ function *lub*

Confinement Flow Model

- $(I, O, \text{confine}, \rightarrow)$
 - $I = (SC_I, \leq_I, \text{join}_I)$
 - O set of entities
 - $\rightarrow: O \times O$ with $(a, b) \in \rightarrow$ (written $a \rightarrow b$) iff information can flow from a to b
 - for $a \in O$, $\text{confine}(a) = (a_L, a_U) \in SC_I \times SC_I$ with $a_L \leq_I a_U$
 - Interpretation: for $a \in O$, if $x \leq_I a_U$, information can flow from x to a , and if $a_L \leq_I x$, information can flow from a to x
 - So a_L lowest classification of information allowed to flow out of a , and a_U highest classification of information allowed to flow into a

Assumptions, *etc.*

- Assumes: object can change security classes
 - So, variable can take on security class of its data
- Object x has security class \underline{x} currently
- Note transitivity *not* required
- If information can flow from a to b , then b dominates a under ordering of policy I :
$$(\forall a, b \in O)[a \rightarrow b \Rightarrow a_L \leq_I b_U]$$

Example 1

- $SC_l = \{ U, C, S, TS \}$, with $U \leq_l C$, $C \leq_l S$, and $S \leq_l TS$
- $a, b, c \in O$
 - $\text{confine}(a) = [C, C]$
 - $\text{confine}(b) = [S, S]$
 - $\text{confine}(c) = [TS, TS]$
- Secure information flows: $a \rightarrow b$, $a \rightarrow c$, $b \rightarrow c$
 - As $a_L \leq_l b_U$, $a_L \leq_l c_U$, $b_L \leq_l c_U$
 - Transitivity holds

Example 2

- SC_l, \leq_l as in Example 1
- $x, y, z \in O$
 - $\text{confine}(x) = [C, C]$
 - $\text{confine}(y) = [S, S]$
 - $\text{confine}(z) = [C, TS]$
- Secure information flows: $x \rightarrow y, x \rightarrow z, y \rightarrow z, z \rightarrow x, z \rightarrow y$
 - As $x_L \leq_l y_U, x_L \leq_l z_U, y_L \leq_l z_U, z_L \leq_l x_U, z_L \leq_l y_U$
 - Transitivity does not hold
 - $y \rightarrow z$ and $z \rightarrow x$, but $y \rightarrow x$ is false, because $y_L \leq_l x_U$ is false

Transitive Non-Lattice Policies

- $Q = (S_Q, \leq_Q)$ is a *quasi-ordered set* when \leq_Q is transitive and reflexive over S_Q
- How to handle information flow?
 - Define a partially ordered set containing quasi-ordered set
 - Add least upper bound, greatest lower bound to partially ordered set
 - It's a lattice, so apply lattice rules!

In Detail ...

- $\forall x \in S_Q$: let $f(x) = \{ y \mid y \in S_Q \wedge y \leq_Q x \}$
 - Define $S_{QP} = \{ f(x) \mid x \in S_Q \}$
 - Define $\leq_{QP} = \{ (x, y) \mid x, y \in S_{QP} \wedge x \subseteq y \}$
 - S_{QP} partially ordered set under \leq_{QP}
 - f preserves order, so $y \leq_Q x$ iff $f(x) \leq_{QP} f(y)$
- Add upper, lower bounds
 - $S_{QP}' = S_{QP} \cup \{ S_Q, \emptyset \}$
 - Upper bound $ub(x, y) = \{ z \mid z \in S_{QP} \wedge x \subseteq z \wedge y \subseteq z \}$
 - Least upper bound $lub(x, y) = \bigcap ub(x, y)$
 - Lower bound, greatest lower bound defined analogously

And the Policy Is ...

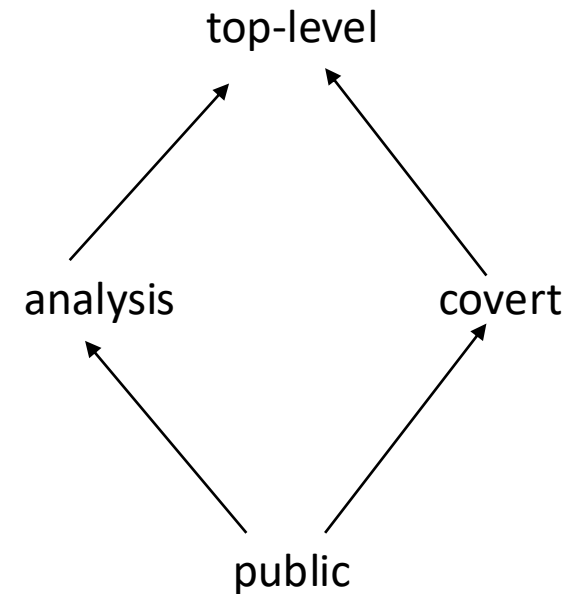
- Now (S_{QP}', \leq_{QP}) is lattice
- Information flow policy on quasi-ordered set emulates that of this lattice!

Nontransitive Flow Policies

- Government agency information flow policy (on next slide)
- Entities public relations officers PRO, analysts A, spymasters S
 - *confine*(PRO) = [public, analysis]
 - *confine*(A) = [analysis, top-level]
 - *confine*(S) = [covert, top-level]

Information Flow

- By confinement flow model:
 - $PRO \leq A, A \leq PRO$
 - $PRO \leq S$
 - $A \leq S, S \leq A$
- Data *cannot* flow to public relations officers; not transitive
 - $S \leq A, A \leq PRO$
 - $S \leq PRO$ is *false*



Transforming Into Lattice

- Rough idea: apply a special mapping to generate a subset of the power set of the set of classes
 - Done so this set is partially ordered
 - Means it can be transformed into a lattice
- Can show this mapping preserves ordering relation
 - So it preserves non-orderings and non-transitivity of elements corresponding to those of original set

Dual Mapping

- $R = (SC_R, \leq_R, join_R)$ reflexive info flow policy
- $P = (S_P, \leq_P)$ ordered set
 - Define *dual mapping* functions $l_R, h_R: SC_R \rightarrow S_P$
 - $l_R(x) = \{x\}$
 - $h_R(x) = \{y \mid y \in SC_R \wedge y \leq_R x\}$
 - S_P contains subsets of SC_R ; \leq_P subset relation
 - Dual mapping function *order preserving* iff
$$(\forall a, b \in SC_R) [a \leq_R b \Leftrightarrow l_R(a) \leq_P h_R(b)]$$

Theorem

Dual mapping from reflexive information flow policy R to ordered set P
order-preserving

Proof sketch: all notation as before

(\Rightarrow) Let $a \leq_R b$. Then $a \in I_R(a)$, $a \in h_R(b)$, so $I_R(a) \subseteq h_R(b)$, or $I_R(a) \leq_P h_R(b)$

(\Leftarrow) Let $I_R(a) \leq_P h_R(b)$. Then $I_R(a) \subseteq h_R(b)$. But $I_R(a) = \{ a \}$, so $a \in h_R(b)$, giving $a \leq_R b$

Information Flow Requirements

- Interpretation: let $confine(x) = [\underline{x}_L, \underline{x}_U]$, consider class \underline{y}
 - Information can flow from x to element of \underline{y} iff $\underline{x}_L \preceq_R \underline{y}$, or $I_R(\underline{x}_L) \subseteq h_R(\underline{y})$
 - Information can flow from element of \underline{y} to x iff $\underline{y} \preceq_R \underline{x}_U$, or $I_R(\underline{y}) \subseteq h_R(\underline{x}_U)$

Revisit Government Example

- Information flow policy is R
- Flow relationships among classes are:

public \leq_R public

public \leq_R analysis

public \leq_R covert

public \leq_R top-level

analysis \leq_R top-level

analysis \leq_R analysis

covert \leq_R covert

covert \leq_R top-level

top-level \leq_R top-level

Dual Mapping of R

- Elements l_R, h_R :

$$l_R(\text{public}) = \{ \text{public} \}$$

$$h_R(\text{public}) = \{ \text{public} \}$$

$$l_R(\text{analysis}) = \{ \text{analysis} \}$$

$$h_R(\text{analysis}) = \{ \text{public}, \text{analysis} \}$$

$$l_R(\text{covert}) = \{ \text{covert} \}$$

$$h_R(\text{covert}) = \{ \text{public}, \text{covert} \}$$

$$l_R(\text{top-level}) = \{ \text{top-level} \}$$

$$h_R(\text{top-level}) = \{ \text{public}, \text{analysis}, \text{covert}, \text{top-level} \}$$

confine

- Let p be entity of type PRO, a of type A, s of type S
- In terms of P (not R), we get:
 - $confine(p) = [\{ public \}, \{ public, analysis \}]$
 - $confine(a) = [\{ analysis \}, \{ public, analysis, covert, top-level \}]$
 - $confine(s) = [\{ covert \}, \{ public, analysis, covert, top-level \}]$

And the Flow Relations Are ...

- $p \rightarrow a$ as $l_R(p) \subseteq h_R(a)$
 - $l_R(p) = \{ \text{public} \}$
 - $h_R(a) = \{ \text{public, analysis, covert, top-level} \}$
- Similarly: $a \rightarrow p, p \rightarrow s, a \rightarrow s, s \rightarrow a$
- But $s \rightarrow p$ is false as $l_R(s) \not\subseteq h_R(p)$
 - $l_R(s) = \{ \text{covert} \}$
 - $h_R(p) = \{ \text{public, analysis} \}$

Analysis

- (S_p, \leq_p) is a lattice, so it can be analyzed like a lattice policy
- Dual mapping preserves ordering, hence non-ordering and non-transitivity, of original policy
 - So results of analysis of (S_p, \leq_p) can be mapped back into $(SC_R, \leq_R, join_R)$

Confinement: Example Problem

- Server balances bank accounts for clients
- Server security issues:
 - Record correctly who used it
 - Send *only* balancing info to client
- Client security issues:
 - Log use correctly
 - Do not save or retransmit data client sends

Generalization

- Client sends request, data to server
- Server performs some function on data
- Server returns result to client
- Access controls:
 - Server must ensure the resources it accesses on behalf of client include *only* resources client is authorized to access
 - Server must ensure it does not reveal client's data to any entity not authorized to see the client's data

Confinement Problem

- Problem of preventing a server from leaking information that the user of the service considers confidential

Total Isolation

- Process cannot communicate with any other process
- Process cannot be observed

Impossible for this process to leak information

- Not practical as process uses observable resources such as CPU, secondary storage, networks, etc.

Example

- Processes p , q not allowed to communicate
 - But they share a file system
- Communications protocol:
 - p sends a bit by creating a file called 0 or 1 , then a second file called $send$
 - p waits until $send$ is deleted before repeating to send another bit
 - q waits until file $send$ exists, then looks for file 0 or 1 ; whichever exists is the bit
 - q then deletes 0 , 1 , and $send$ and waits until $send$ is recreated before repeating to read another bit

Covert Channel

- A path of communication not designed to be used for communication
- In example, file system is a (storage) covert channel

Rule of Transitive Confinement

- If p is confined to prevent leaking, and it invokes q , then q must be similarly confined to prevent leaking
- Rule: if a confined process invokes a second process, the second process must be as confined as the first

Lipner's Notes

- All processes can obtain rough idea of time
 - Read system clock or wall clock time
 - Determine number of instructions executed
- All processes can manipulate time
 - Wait some interval of wall clock time
 - Execute a set number of instructions, then block

Approach: Isolation

- Constrain process execution in such a way it can only interact with other entities in a manner preserving isolation
 - Hardware isolation
 - Virtual machines
 - Library operating systems
 - Sandboxes
- Modify program or process so that its actions will preserve isolation
 - Program rewriting
 - Compiling
 - Loading

Hardware Isolation

- Ensure the hardware is disconnected from any other system
 - This includes networking, including wireless
- Example: SCADA systems
 - 1st generation: serial protocols, not connected to other systems or networks; no security defenses needed, focus being on malfunctions
 - 2nd generation: serial networks connected to computers not connected to Internet
 - 3rd generation: TCP/IP protocol running on networks connected to Internet; need security defenses for attackers coming in over Internet
- Example: electronic voting systems
 - Physical isolation protects systems from attackers changing votes remotely
 - Required in many U.S. states, such as California: never connect them to any network

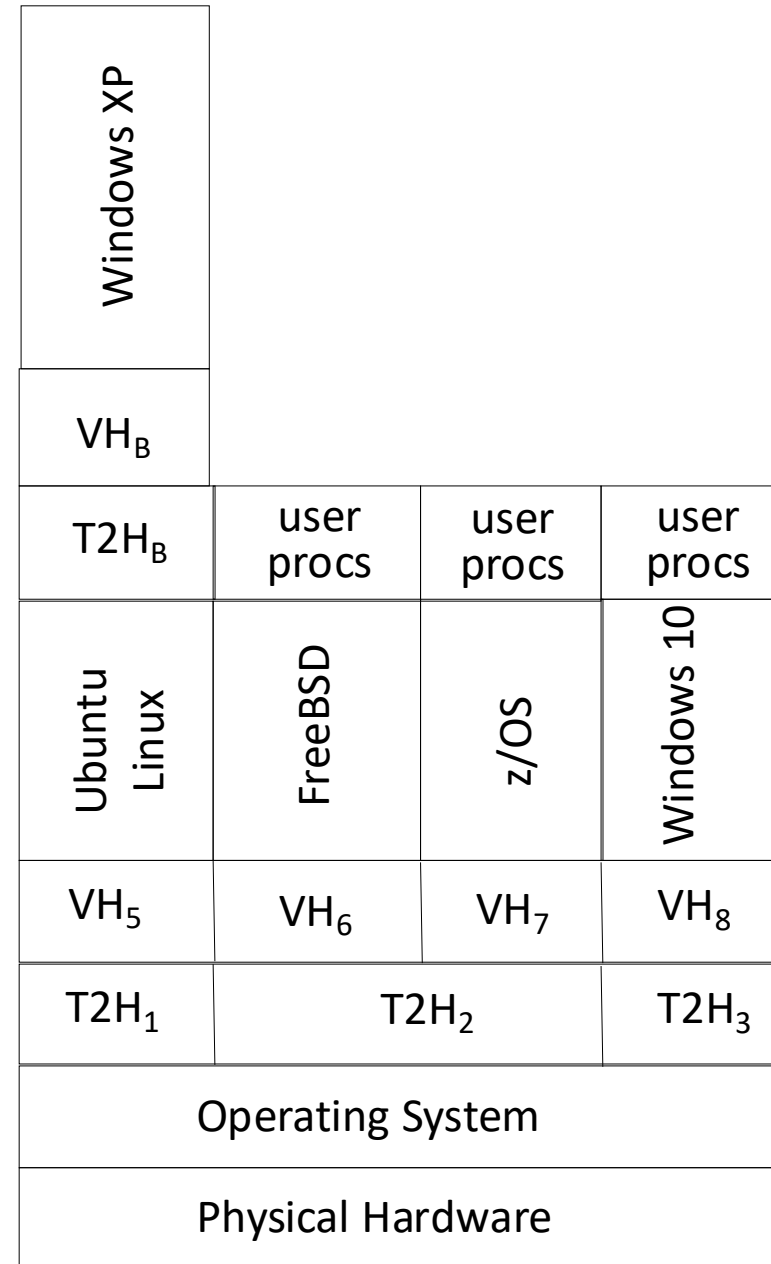
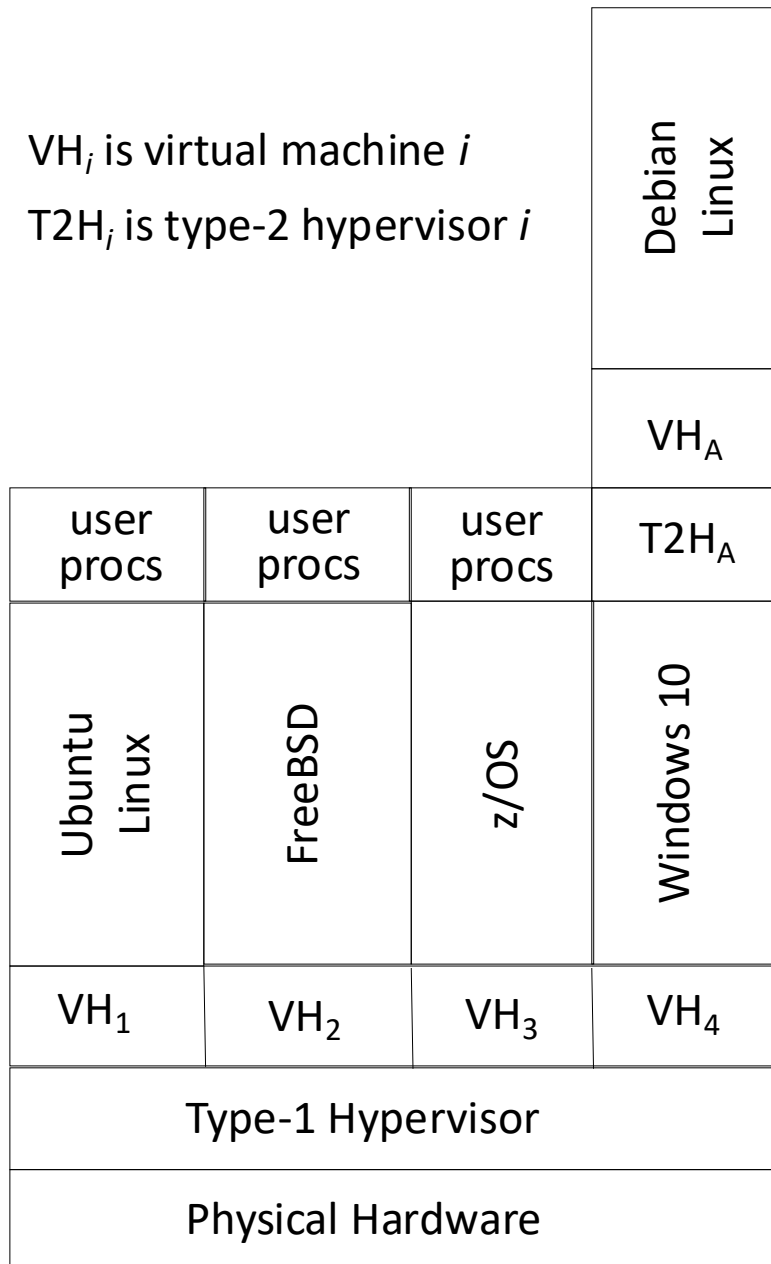
Virtual Machine

- Program that simulates hardware of a machine
 - Machine may be an existing, physical one or an abstract one
 - Uses special operating system, called *virtual machine monitor (VMM)* or *hypervisor*, to provide environment simulating target machine
- Types of virtual machines
 - Type 1 hypervisor: runs directly on hardware
 - Type 2 hypervisor: runs on another operating system
- Existing OSes do not need to be modified
 - Run under VMM, which enforces security policy
 - Effectively, VMM is a security kernel

What Is It?

- *Virtual machine monitor (VMM) or hypervisor* virtualizes system resources
 - Provides interface to give each program running on it the illusion that it is the only process on the system and is running directly on hardware
 - Provides illusion of contiguous memory beginning at address 0, a CPU, and secondary storage to *each* program
- *Type-1 hypervisor* runs directly on the hardware
- *Type-2 hypervisor* runs as a process on a regular operating system

VH_i is virtual machine i
 $T2H_i$ is type-2 hypervisor i



Privileged Instructions

1. VMM runs VM with operating system o , which is running process p
 - p tries to read, so privileged operation traps to hardware
2. VMM invoked, determines trap occurred in VM
 - VMM updates state of VM to make it look like hardware invoked o directly, so o tries to read, causing trap
3. VMM does read
 - Updates VM to make it seem like o did read
 - Transfers control to o

Privileged Instructions

4. o tries to switch context to p , causing another trap
5. VMM updates VM running o to make it appear o did context switch successfully
 - Transfers control to o , which (as o apparently did a context switch to p) returns control to p

Privileged Instructions

